cussing not only scientific concepts but religious ones in order to see where there may be real conflict and where consonance may exist. Religious believers may come to see how their faith can be expressed in theologies that are open to scientific understandings of the world, and scientists may come to see the openness of their scientific understandings to religious belief. The possibility of such mutual understanding is shown by the existence of many people who have been both scientists and religious believers, such as James Clerk Maxwell.

Of course there are religious believers who insist on shielding their faith from genuine encounter with science, just as there are scientists whose antireligious prejudices make discussion impossible. Theological as well as scientific illiteracy can be a problem. But it is encouraging that the science–theology dialogue is growing at a healthy rate, as shown by the number of organizations, meetings and publications devoted to it. If the results of what has so far been primarily an academic discussion can be made accessible to laypeople, they may help to alleviate the problem Smith has identified.

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7/91

Electron–Photon Analogy Analyzed

In the May 1991 issue (page 32), Sajeev John makes many interesting statements about light localization. However, the analogy he draws between electrons and photons has several flaws.

The analogy between Schrödinger's equation

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 + V(\mathbf{x})\right]\psi(\mathbf{x}) = E\,\psi(\mathbf{x}) \quad (1)$$

and the wave equation for $\mathbf{E}$ in a dielectric medium

$$-\nabla^2\mathbf{E} + \nabla(\nabla\cdot\mathbf{E}) - \frac{\omega^2}{c^2}\epsilon_{\text{fluct}}(\mathbf{x})\,\mathbf{E} = \frac{\omega^2}{c^2}\epsilon_0\mathbf{E} \quad (2)$$

is incomplete, in that the $\nabla(\nabla\cdot\mathbf{E})$ term has no counterpart in Schrödinger's equation, or in *any* scalar wave equation. John identifies $(\omega^2/c^2)\epsilon_{\text{fluct}}$ as the "potential" and $(\omega^2/c^2)\epsilon_0$ as the "energy," but does not address the question of where the $\nabla(\nabla\cdot\mathbf{E})$ term belongs. Since $\nabla\cdot\mathbf{D} = 0$, $\nabla\cdot\mathbf{E}$ is the polarization charge density and vanishes if $\epsilon_{\text{fluct}} = 0$ everywhere. Hence $\nabla\cdot\mathbf{E}$ would rightly belong with the "potential" term, if one *had* to make the analogy. However, that is not necessary. Equation 2 can be written as

$$\nabla\times\nabla\times\mathbf{E} = \frac{\omega^2}{c^2}\epsilon(\mathbf{x})\,\mathbf{E} \quad (3)$$

which is a *generalized* Hermitian eigenproblem.[1] The orthogonality relation for the eigenfunctions is

$$\int \epsilon(\mathbf{x})\,\mathbf{E}_n^*\cdot\mathbf{E}_m\,d\mathbf{x} = \text{const}\cdot\delta_{nm}$$

that is, with $\epsilon(\mathbf{x})$ as the *weight function*. Formally, $\epsilon(\mathbf{x})$ can be viewed as the "density of space." Just as the curvature of space bends or "scatters" light, so do "density fluctuations" of space.

John further writes: "ω multiplies the scattering potential $\epsilon_{\text{fluct}}(x)$. . . . [L]owering the photon energy . . . leads to a complete disappearance of scattering." If this were true, we could drop the "potential" in equation 2 as $\omega\to 0$. We would then end up with an equation for $\mathbf{E}$ in a homogeneous medium and the conclusion that the effective long-wavelength dielectric constant $\epsilon_{\text{eff}} = \epsilon_0$ for any inhomogeneous medium. This conclusion would be incorrect.[2]

What is missing in John's depiction is that the "energy" too vanishes as $\omega\to 0$. The ratio of the two, $\epsilon_{\text{fluct}}(\mathbf{x})/\epsilon_0$, is independent of ω. Hence discarding the "potential" while retaining the "energy" is not justified in any frequency regime. Furthermore, $\|\epsilon_{\text{fluct}}\|/\epsilon_0$, which is a measure of how large the "potential" is relative to "energy," is not necessarily small. Although Rayleigh scattering from a *single* dielectric object vanishes as $\omega\to 0$, in the case of *multiple* scattering even an infinitesimal amplitude for each event would, in general, yield a finite effect.

It is clear that periodic dielectric or paramagnetic structures would yield interesting physics. Any environment whose Green's function differs from that of free space would alter quantum electrodynamics, as noted by Asim O. Barut and Jonathan P. Dowling.[3] However, there are experimental and computational[1] difficulties that need to be overcome before definitive conclusions can be drawn.

References

1. H. S. Sözüer, J. W. Haus, R. Inguva, submitted to Phys. Rev. B.
2. See, for example, W. Lamb, D. M. Wood, N. W. Ashcroft, Phys. Rev. B **21**, 2248 (1980), and references therein.

3. A. O. Barut, J. P. Dowling, Phys. Rev. A **36**, 649 (1987).

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John replies: H. Sami Sözüer, Ramarao Inguva and Joseph W. Haus criticize my analogy between Schrödinger's equation and the electromagnetic wave equation. As with any analogy, one must take care when discussing properties for which the analogy was not intended. I see no flaw in this analogy as far as the conclusions drawn in my article "Localization of Light" are concerned.

The purpose of the analogy was only to elucidate the nature of electromagnetic wave scattering. Scattering in this context is defined as an event that is observed on some scale comparable to or greater than the wavelength λ of the radiation. To make use of the analogy, microscopic polarization effects that occur on much shorter length scales must already be absorbed into the average dielectric constant ϵ_0, and the electric field vector **E** must be interpreted as a macroscopic field that is coarse grained over the scale of the wavelength. If one instead interprets **E** to be the microscopic electric field, this quantity exhibits strong fluctuations on scales short compared with λ as a result of random polarization effects of very small dielectric microstructures. This, however, *does not* lead to significant scattering of the macroscopic electric field but determines the effective dielectric constant ϵ_{eff}, which, as Sözüer and his colleagues point out, is different from the microscopic ϵ_0. The weak scattering of the macroscopic electric field is nevertheless obtained correctly from the analogy. The result that the transport mean free path l^* diverges as $\omega^{-(d+1)}$ contains both the facts that the "potential" vanishes as ω^2 and that the "energy" vanishes, in contradiction to the statement by Sözüer and his coauthors that the analogy misses this. The vanishing of the "energy" gives a density of final states into which scattering can take place that vanishes as ω^{d-1}. This result is crucial in obtaining the above value for l^*.

Sözüer and his coauthors have not found a flaw in the analogy in the context in which it is used but have shown how an inappropriate application of the analogy can lead to erroneous conclusions.

Finally, Sözüer and his colleagues make some remarks concerning quantum electrodynamics in confined geometries following the work of Daniel Kleppner and collaborators on small metal cavities comparable in size to the wavelength of radiation. Dielectric materials with photonic bandgaps offer two advantages over metallic systems. First of all, dissipation is more easily controlled in dielectric systems. A single localized state in a photonic bandgap acts much like a high-Q optical cavity. A dielectric material with an absorption length $l_{\text{abs}} > 3$ km has an absorption time $\tau > 10^{-5}$ sec. At an optical frequency $\nu \sim 10^{15}$ Hz, this gives a quality factor $Q \sim \nu\tau$ exceeding that available in most metals. Second, dielectric systems can be produced as bulk materials and are not limited to very small sizes. This offers the possibility of observing many new phenomena, as my article described. For the case of the photon–atom bound state, the primary experimental difficulty is nonradiative relaxation if the atom is embedded in the solid fraction of the material. This problem, however, may be overcome by Doppler cooling of atoms into the cylindrical void regions of the material. In this case, I argued in my article, the lifetime τ could be on the scale of minutes given the same absorption length of a few kilometers. A second possibility is that these nonradiative effects may be overwhelmed by resonance dipole–dipole hopping conduction of the bound photon from one atom to another. In this case the photon would bind to some cluster of atoms, and the nonradiative contribution to the level width would exhibit motional narrowing. Any further discussion concerning the experimental observability of these effects is of course most welcome.

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11/91

Corrections

February, page 38—Erich Bloch is a Distinguished Fellow of the Council on Competitiveness, not the White House Council on Competitiveness.

December, page 18—Jacques des Cloizeaux is not affiliated with Orsay; he has had a long career in the Service de Physique Théorique of Saclay. ■