Superconducting quasiparticle lifetimes due to spin-fluctuation scattering

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Superconducting quasiparticle lifetimes associated with spin-fluctuation scattering are calculated. A Berk-Schrieffer interaction with an irreducible susceptibility given by a BCS form is used to model the quasiparticle damping due to spin fluctuations. Results are presented for both *s*-wave and *d*-wave gaps. Also, quasiparticle lifetimes due to impurity scattering are calculated for a *d*-wave superconductor.

In the traditional low-temperature superconductors, the dominant dynamic quasiparticle relaxation processes involve the electron-phonon interaction. In these materials there are inelastic-scattering events in which a quasiparticle emits or absorbs a phonon, as well as events in which two quasiparticles recombine to form a pair or in which a pair is broken into two quasiparticles with the emission or absorption of a phonon, respectively. In some materials with large Debye energies, dynamic electron-electron scattering processes can play a role at low temperatures. In the cuprates, however, the dynamic electron-electron scattering processes are enhanced by the existence of strong short-range antiferromagnetic spin fluctuations. We have previously found[1] that the existence of such spin fluctuations can account for the temperature dependence of the nuclear-spin-relaxation time T_1. Below T_c the temperature dependence of T_1 could be fit by using a random-phase-approximation form for $\chi(\mathbf{q},\omega)$ with an irreducible susceptibility given by the BCS susceptibility with a *d*-wave gap. Here we extend this approach to determine the scattering and recombination quasiparticle lifetimes for both *s*- and *d*-wave gaps. We compare our results with recent transport lifetimes reported by Bonn and co-workers.[2] We also examine the influence of impurity scattering on the low-temperature quasiparticle lifetime for the *d*-wave case.

Our approach is similar to the quasiparticle lifetime calculations of Kaplan *et al.*[3] for the low-temperature superconductors except that we assume that antiferromagnetic spin fluctuations rather than phonons provide the dominant relaxation mechanism. Specifically, we consider a Hubbard model on a two-dimensional lattice with a near-neighbor hopping t and an on-site Coulomb interaction U. We approximate the spin-fluctuation interaction $V(\mathbf{q},\omega)$ which enters the self-energy by

$$V(\mathbf{q},\omega)\cong\tfrac{3}{2}\bar{U}/[1-\bar{U}\chi_0^{\mathrm{BCS}}(\mathbf{q},\omega)]\ . \tag{1}$$

Here $\bar{U}$ is a reduced effective interaction chosen along with the filling $\langle n\rangle=\langle n_{i\uparrow}+n_{i\downarrow}\rangle$ to adjust the strength of the antiferromagnetic spin fluctuations, and χ_0^{BCS} is the BCS susceptibility

$$\begin{aligned}\chi_0^{\mathrm{BCS}}(\mathbf{q},\omega)=\int\frac{d^2p}{(2\pi)^2}\Bigg\{&\frac{1}{2}\left[1+\frac{\varepsilon_{\mathbf{p}+\mathbf{q}}\varepsilon_{\mathbf{p}}+\Delta_{\mathbf{p}+\mathbf{q}}\Delta_{\mathbf{p}}}{E_{\mathbf{p}+\mathbf{q}}E_{\mathbf{p}}}\right]\frac{f(E_{\mathbf{p}+\mathbf{q}})-f(E_{\mathbf{p}})}{\omega-(E_{\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}})+i0^+}\\&+\frac{1}{4}\left[1-\frac{\varepsilon_{\mathbf{p}+\mathbf{q}}\varepsilon_{\mathbf{p}}+\Delta_{\mathbf{p}+\mathbf{q}}\Delta_{\mathbf{p}}}{E_{\mathbf{p}+\mathbf{q}}E_{\mathbf{p}}}\right]\frac{1-f(E_{\mathbf{p}+\mathbf{q}})-f(E_{\mathbf{p}})}{\omega+(E_{\mathbf{p}+\mathbf{q}}+E_{\mathbf{p}})+i0^+}\\&+\frac{1}{4}\left[1-\frac{\varepsilon_{\mathbf{p}+\mathbf{q}}\varepsilon_{\mathbf{p}}+\Delta_{\mathbf{p}+\mathbf{q}}\Delta_{\mathbf{p}}}{E_{\mathbf{p}+\mathbf{q}}E_{\mathbf{p}}}\right]\frac{f(E_{\mathbf{p}+\mathbf{q}})+f(E_{\mathbf{p}})-1}{\omega-(E_{\mathbf{p}+\mathbf{q}}+E_{\mathbf{p}})+i0^+}\Bigg\}\ .\end{aligned} \tag{2}$$

This has the usual coherence factors with $E_{\mathbf{p}}=\sqrt{\varepsilon_{\mathbf{p}}^2+\Delta_{\mathbf{p}}^2}$ and $\varepsilon_{\mathbf{p}}=-2t(\cos p_x+\cos p_y)-\mu$. Results will be presented for both an *s*-wave gap $\Delta_s=\Delta_0(T)$ and a *d*-wave gap $\Delta_d(\mathbf{p})=(\Delta_0(T)/2)(\cos p_x-\cos p_y)$. In the following, we will use the parameters $\bar{U}/t=2$ and $\langle n\rangle=0.85$, which are similar to those used previously in our analysis of the NMR data.[1] We also assume that $\Delta_0(T)$ has the usual mean-field temperature dependence.

With the interaction given by Eq. (1) and following the approach described in Ref. 3, we find that the inverse lifetime for a quasiparticle of energy ω and momentum $\mathbf{p}$ is given by

$$\begin{aligned}\tau^{-1}(\mathbf{p},\omega)=\int\frac{d^2p'}{(2\pi)^2}\Bigg\{&\int_0^{\omega-|\Delta_{\mathbf{p}'}|}d\Omega\,\mathrm{Im}V(\mathbf{p}-\mathbf{p}',\Omega)\delta(\omega-\Omega-E_{\mathbf{p}'})\left[1+\frac{\Delta_{\mathbf{p}}\Delta_{\mathbf{p}'}}{\omega(\omega-\Omega)}\right][n(\Omega)+1][1-f(\omega-\Omega)]\\&+\int_{\omega+|\Delta_{\mathbf{p}'}|}^{\infty}d\Omega\,\mathrm{Im}V(\mathbf{p}-\mathbf{p}',\Omega)\delta(\Omega-\omega-E_{\mathbf{p}'})\left[1-\frac{\Delta_{\mathbf{p}}\Delta_{\mathbf{p}'}}{\omega(\Omega-\omega)}\right][n(\Omega)+1]f(\Omega-\omega)\\&+\int_0^{\infty}d\Omega\,\mathrm{Im}V(\mathbf{p}-\mathbf{p}',\Omega)\delta(\Omega+\omega-E_{\mathbf{p}'})\left[1+\frac{\Delta_{\mathbf{p}}\Delta_{\mathbf{p}'}}{\omega(\Omega+\omega)}\right]n(\Omega)[1-f(\Omega+\omega)]\Bigg\}\ .\end{aligned} \tag{3}$$

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Here a quasiparticle renormalization factor has been absorbed into V, and $n(\Omega)$ and $f(\omega)$ are the usual Bose and Fermi factors. The first and third terms give contributions to the quasiparticle scattering rate τ_s^{-1} arising from the emission and absorption of a spin-fluctuation, respectively. The second term is the recombination rate τ_r^{-1} corresponding to a process in which a quasiparticle recombines with another quasiparticle to form a pair with the excess energy emitted as a spin fluctuation. Note that the signs in the coherence factors for the antiferromagnetic spin-fluctuation interaction are opposite to those for the phonon case.

As seen in neutron-scattering experiments,[4,5] when the system goes superconducting, the low-frequency $\omega < 2\Delta$ spin-fluctuation spectral weight is reduced over most of the Brillouin zone. It follows from Eq. (3) that this will lead to a decrease in the quasiparticle decay rate. Nuss *et al.*[6] and Littlewood and Varma[7] suggested that this was responsible for the peak observed in the far-infrared conductivity rather than a coherence factor.[8] This may be contrasted with the phonon case in which the low-frequency phonon spectral weight is essentially unchanged in the superconducting state.

The momentum integrations in Eqs. (2) and (3) cover the Brillouin zone. They are carried out by dividing up the momentum space into a grid of small squares and evaluating the integrands at the center of each square. The square size is then reduced until the finite-size effects become negligible. The exception to this procedure is the evaluation of $\mathrm{Im}\chi_0(\mathbf{q},\omega)$. Since this involves an integral over a δ function, a slightly different procedure is required. For this integration, the small squares covering the momentum space are further divided onto four triangular regions each. The argument of the δ function involved in the integral is then evaluated at the three vertices of each triangle. This allows the argument to be replaced by a linear approximation of itself within a given triangle. Once this replacement is done, the integral of the δ function over the triangular region may be done analytically. Repeating this procedure over the whole zone completes the momentum integration.

Results for the quasiparticle decay rate obtained from Eq. (3) for s- and d-wave gaps with $2\Delta_0(0)/kT_c = 6$ are shown in Figs. 1 and 2. Here we have taken $T_c = 0.2t$ in order to examine the behavior of the decay rate at small values of T/T_c. We have normalized $\tau(T)$ by its value at T_c.[9] For both cases we have set $\mathbf{p}$ on the Fermi surface with $p_x = p_y$. For the d-wave gap this corresponds to looking at a quasiparticle in the region of the node where the gap vanishes. In this case we have set ω equal to the thermal energy kT. This is representative of a typical quasiparticle energy and momentum at lower temperatures where the nodal regions of the Fermi surface are predominantly occupied. For the s-wave gap below T_c we have set $\omega = \Delta_0(T)$.

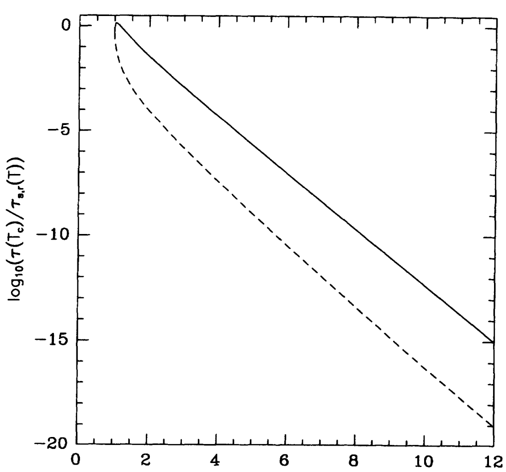

FIG. 1. Plot of $\log_{10}(\tau(T_c)/\tau_{r,s}(T))$ versus T_c/T for an s-wave gap with $2\Delta_0(0)/kT_c = 6$ and $T_c = 0.2t$. The solid line shows the quasiparticle scattering rate τ_s^{-1}. The dashed line shows the recombination rate τ_r^{-1}. At low temperatures both curves fall as $\exp(-\Delta/T)$.

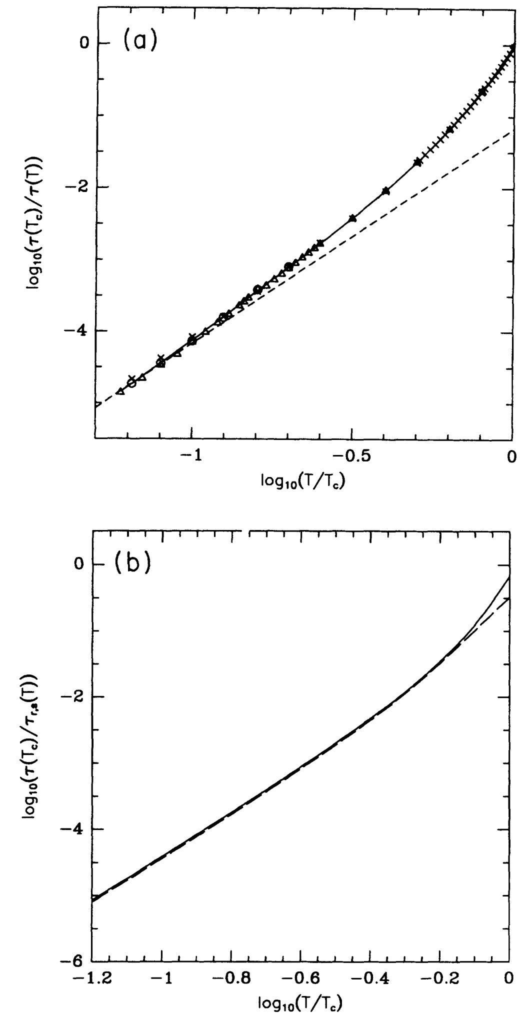

FIG. 2. (a) Plot of $\log_{10}(\tau(T_c)/\tau(T))$ versus $\log_{10}(T/T_c)$ for a d-wave gap with $2\Delta_0(0)/kT_c = 6$ and $T_c = 0.2t$. At low temperatures $\tau^{-1}(\tau)$ varies as $(T/T_c)^3$. The dashed line displays a slope of three on the log-log plot as a guide to the eye. The symbols represent τ^{-1} values calculated using different momentum lattice sizes: 512×512 (circles), 256×256 (triangles), and 128×128 (crosses). (b) Plot showing separately the scattering rate τ_s^{-1} (solid line) and the recombination rate τ_r^{-1} (dashed line).

In Fig. 1 we show the temperature dependence of the quasiparticle scattering and recombination rates for an s-wave gap on a semilog plot. This clearly shows that both rates decrease exponentially as $e^{-\Delta/T}$ due to the opening of a gap in the spin-fluctuation spectrum. In Figs. 2(a) and 2(b) we show similar results for the d-wave case on a log-log scale. This shows that at very low temperatures the relaxation rates for a d-wave gap have a T^3 dependence associated with the available phase space for scattering in the nodal regions. The scattering and recombination rates in the nodal region are comparable over most of the temperature range shown, due to the small quasiparticle frequency. In order to show the magnitude of the finite lattice size error in this calculation, Fig. 2(a) includes points showing the τ^{-1} values calculated using a few different momentum lattices sizes.

Using microwave surface-resistance and penetration-depth data, Bonn and co-workers[2] find that the real part of the conductivity of $YBa_2CuO_{6.95}$ exhibits a broad peak around 40 K, which has a height 10–20 times the value of the conductivity at T_c. Within the framework of a generalized two-fluid model in which the conductivity of the normal fluid is modeled by a Drude form, they extract a transport lifetime and find that the inverse of this lifetime decreases rapidly with decreasing temperature as the temperature drops below T_c. At temperatures below $T_c/2$ they find that this lifetime reaches a temperature-independent limiting value. Semilog plots of the quasiparticle decay rates for s- and d-wave gaps are shown in Figs. 3(a) and 3(b) along with data from Ref. 2. The d-wave gap results with $2\Delta_0(0)/kT_c \sim 6$ to 8 appear to roughly follow the microwave results above $T/T_c = 0.5$.

Thus we find that a spin-fluctuation interaction with χ_0^{BCS} calculated within a BCS framework using a d-wave gap yields a quasiparticle decay rate which decreases rapidly with decreasing temperature below T_c. Well below T_c this decay rate exhibits a T^3 temperature dependence. Above $T_c/2$ this decay rate is in rough agreement with the observed temperature dependence of the inverse transport lifetime $\tau_{\mathrm{tr}}(T)$ obtained by Bonn and co-workers.[2] A similar calculation using an s-wave gap does not appear to provide as satisfactory a fit. This suggests that, assuming the transport lifetime is indeed driven by spin-fluctuation scattering, these fluctuations are suppressed, but apparently not gapped, below T_c.

Bonn and co-workers[2] suggest that the limiting value that the transport lifetime reaches at low temperatures may be due to impurity scattering. To investigate this possibility, we have calculated the impurity scattering rate for a d-wave superconductor with model parameters as given above. In the dilute impurity concentration limit the quasiparticle scattering rate is given by[10]

$$\tau_{\mathrm{imp}}^{-1}(\omega) = -2\,\mathrm{Im}\Sigma_0(\omega) \tag{4}$$

with the self-energy $\Sigma_0(\omega)$

$$\Sigma_0(\omega) = \Gamma G_0(\omega)/[c^2 - G_0(\omega)^2] \; . \tag{5}$$

Here $\Gamma = n_i/(\pi N(0))$, $c = \cot\delta_0$, and

$$G_0(\omega) = \frac{1}{\pi N(0)} \int \frac{d^2p}{(2\pi)^2} \frac{\omega - \Sigma_0(\omega)}{[\omega - \Sigma_0(\omega)]^2 - E_p^2} \; , \tag{6}$$

where n_i is the impurity concentration, $N(0)$ is the normal phase density of states, and δ_0 is the scattering phase shift.[11]

We have solved these equations for the case of a dilute concentration of impurities such that $\Gamma/\Delta_0(0) = 10^{-3}$. The results are plotted in Fig. 4 for several values of the phase shift parameter c. Since the impurity scattering rate is strongly dependent on the quasiparticle frequency, a weighted average of this scattering rate will enter the transport lifetime. For illustrative purposes we have set $\omega = T$ in the impurity scattering rate and added this to the spin-fluctuation driven quasiparticle decay rate shown previously. In Fig. 5 this combined relaxation rate is compared to the results from Ref. 2. It appears that for any choice of the phase shift parameter we find more structure at low temperatures in the impurity scattering rate than obtained in the phenomenological analysis of Ref. 2.

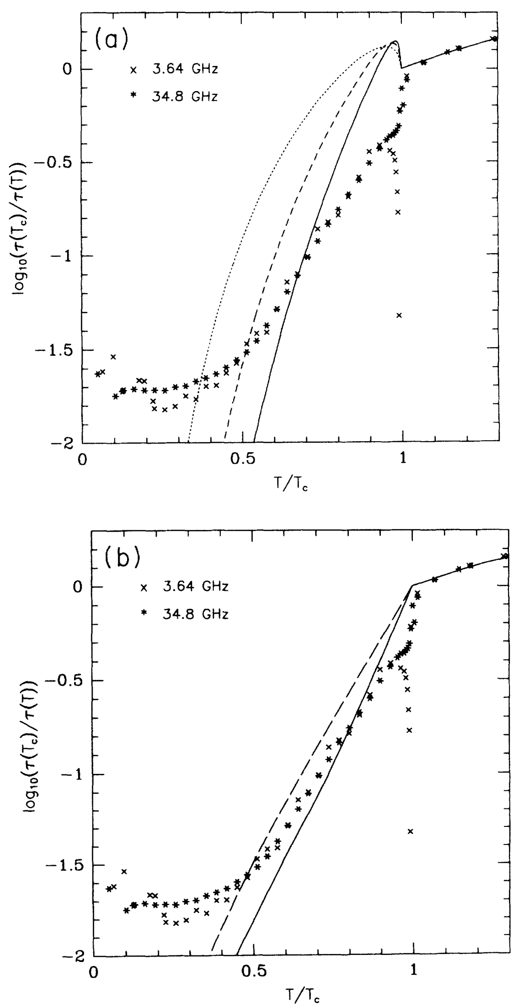

FIG. 3. Plot of $\log_{10}(\tau(T_c)/\tau(T))$ versus T/T_c, where $T_c = 0.1t$, for (a) an s-wave gap with $2\Delta_0(0)/kT_c = 4$ (dotted), 6 (dashed), and 8 (solid) and (b) a d-wave gap with $2\Delta_0(0)/kT_c = 6$ (dashed) and 8 (solid). The symbols are data from Ref. 2.

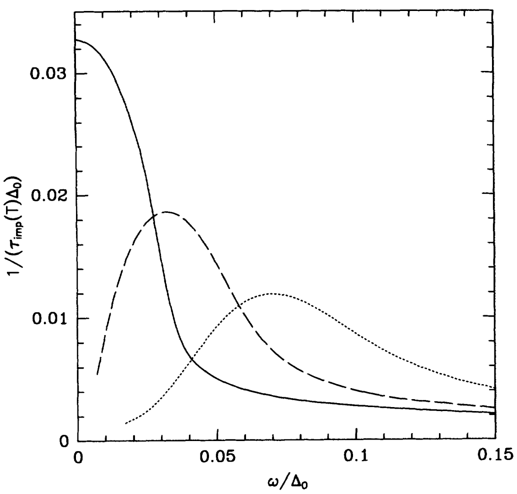

FIG. 4. Plot of impurity scattering rate $1/(\tau_{\rm imp}(\omega)\Delta_0)$ versus frequency ω/Δ_0 at $T=0$ for $\Gamma/\Delta_0(0)=10^{-3}$. This scattering rate is shown various phase shift parameter values: $c=0$ (the unitary limit, solid line), $c=0.1$ (dashed line), and $c=0.2$ (dotted line).

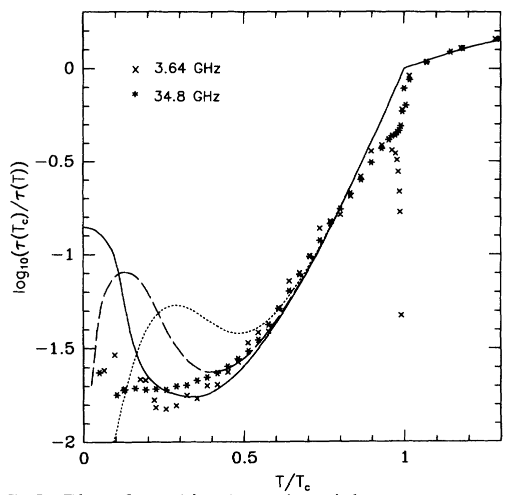

FIG. 5. Plot of combined quasiparticle decay rates due to impurity and spin-fluctuation scattering versus temperature compared with results from Ref. 2. Here we have used $T_c=0.1t$, $2\Delta_0(0)/kT_c=8$, and $\Gamma/\Delta_0(0)=10^{-3}$. The three curves shown are for various phase shift parameter values: $c=0$ (the unitary limit, solid line), $c=0.1$ (dashed line), and $c=0.2$ (dotted line).

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[8]B. W. Statt and A. Griffin [Phys. Rev. B **46**, 3199 (1992)], on the other hand, suggested that a uniform suppression below T_c of the spin-fluctuation spectral weight at all frequencies can account for this peak.

[9]Previous calculations using the interaction given by Eq. (1) have shown that spin fluctuations lead to the right order of magnitude for the quasiparticle relaxation rate in the normal state with $\tau^{-1}(T_c)\sim T_c$ [S. Wermbter and L. Tewordt, Phys. Rev. B **43**, 10 530 (1991), and N. Bulut, H. Morawitz, and D. J. Scalapino (unpublished)]. Furthermore, this same strength of interaction accounts for the correct size of the spin-lattice relaxation rate[1] $T_1^{-1}(T_c)$.

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[11]Note that to obtain the above equations for the impurity scattering rate we must assume both that the average of the superconducting gap $\Delta_{\rm p}$ over the Fermi surface vanishes and that the system has particle-hole symmetry (Ref. 10). This second assumption does not hold for the tight-binding band used in our model. However, we are only interested in the impurity scattering at temperatures and frequencies small compared to the superconducting gap Δ_0. At such a small energy scale particle-hole symmetry is approximately obeyed, and corrections due to the lack of such symmetry will be small.