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Electroweak Symmetry Breaking in Low-Scale Quantum Gravity

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Abstract

We discuss the implications of low-scale quantum gravity on electroweak symmetry breaking by computing one-loop gravitational corrections to the Higgs potential. We conclude that these radiative corrections are quite large after summing over the graviton and dilaton Kaluza-Klein towers, and there is no possibility of electroweak symmetry breaking unless one allows for the complex vacuum expectation value for the Higgs field.

1. Introduction

Unification of gravity with other forces of Nature requires that the world has more than three spatial dimensions. That these extra spatial dimensions have not been observed has the conventional explanation, namely, that they are compactified with tiny radii of the order of Planck length $\ell_{Pl} \sim 10^{-33}$ cm [1]. Thus, probing these extra dimensions require energies on the order of Planck mass $M_{pl} = 2.4 \times 10^{18}$ GeV, which is far beyond the reach in the framework of existing collider paradigm. In this sense, the Planckian energy M_{pl} is the scale at which all four forces of the Nature become unified through the slope of the Regge trajectory α' and Newton's constant G_N in a string-theoretic picture. Thus, string theory, in particular supergravity, unifies the forces of Nature in the far ultraviolet-end of the relevant energy scales.

However, M_{pl} is not the unique mass scale of Nature. In fact, in the infrared-end of the spectrum there exists the scale of electroweak symmetry breaking, $M_{EW} \sim 10^2$ GeV. This is the scale where the electroweak symmetry is spontaneously broken, through which the experimentally confirmed spectra of ferminos and vector bosons are generated. The Standard Model of electroweak interactions (SM) has been found to agree with all experiments to excellent accuracy [2]. However, it contains not only the vector bosons and fermions but also a spinless boson, the Higgs boson h, whose existence has not been experimentally confirmed yet.

Whatever value the experimental observation might yield someday, the mass of the Higgs boson, m_h should be proportional to the weak scale, that is, $m_h \sim M_{EW}$ by the physical consistency of the model. However, if one computes simple one-loop radiative corrections to m_h it turns out that the loop amplitude diverges quadratically, and necessarily, by $\delta m_h^2 \sim M_{pl}^2$. This is what we call the "hierarchy problem" of the SM, that is, the scalar sector of the theory is not stable under radiative corrections: $\delta m_h^2/m_h^2 \sim 10^{16}$. Such a hierarchy problem does not arise in the fermionic sector, thanks to chiral symmetry.

The stability problem of the SM has a long standing solution in terms of the supersymmetric models [3] where the quadratic divergence in δm_h^2 is cancelled by the corresponding superpartner's contribution. Since supersymmetry, if any, is a badly broken symmetry of Nature, the sole constraint on the breaking scale comes from the hierarchy problem; that is the mass-squared difference between the superpartners is to be tuned to be proportional to the weak scale. This then entails the expectation that supersymmetry will break around TeV scales for a successful phenomenology of the Higgs sector [4]. Despite all these good news, however, supersymmetric theories do have their own hierarchy problem, that is, the so-called μ-problem. The μ parameter, the Dirac mass term for the fermionic Higgs fields, can be anywhere between M_{EW} and M_{pl}, as there is no symmetry whatsoever to say it is at M_{EW} scale. It has been suggested that the μ-problem could be evaded if either the Higgs sector or both Higgs sector and the gauge structure is extended. Though the former mechanism has been ruled out by cosmological reasons [5], the latter still works according to the present-day collider bounds [6].

Very recently it has been suggested that 'new physics', into which the SM is expected to be embedded around TeV energies could be of purely gravitational origin [7]. The prime assumption concerning the supergravity models is that the Newtonian gravity will be valid down to l_{pl}; however, gravitational interactions have been accurately tested only down to $\sim$ 1cm so far. This fact opens an important possibility that the gravitational laws could deviate from the Newtonian form at the level of $\sim$ 1 cm and below. Possible negative searches at the colliders can only constrain this distance down to smaller values but can never exclude it completely unless all length scales down to l_{pl} are probed. This appealing phenomenological observation implies that the extra spatial dimensions, which are necessary for making gravity strong enough to unify with other forces, can be as large $\sim$ 1 cm, which is thirty three orders of magnitude larger than l_{pl}. It is clear that if gravity becomes strong enough around $\sim$ TeV, then it replaces the so-called 'new physics' without any need for supersymmetry. In this case, the quadratic divergence in no way causes a hierarchy problem, since the ultraviolet cut-off for the SM is at weak scale itself.

In this work, the main concern will be on the modifications in the electroweak Higgs potential, and the electroweak symmetry breaking mechanism in the presence of large ($\sim \mathcal{O}$ (cm)) extra dimensions. In particular, analysis of the mass generation itself under quantum gravitational effects may shed light on the nature of the symmetry breaking.

In Sec. 2 we discuss relevant aspects of low-scale quantum gravity, in particular, the compactification mechanism for the extra dimensions. Then we analyze in detail the modifications in the electroweak Higgs potential and the electroweak symmetry breaking itself. In Sec. 3 we summarize and discuss our results.

2. Quantum Gravity and Higgs Potential

As a warm-up exercise, consider a point-like, electrically charged, massive particle located at the origin of three-dimensional space. Moreover, suppose that, except for the close vicinity of the particle, the entire space is filled up by a perfect conductor having infinite conductivity. The existence of this electrically conducting surrounding medium simulates the actual situation one would have if the particle were in free space. In the latter situation, electromagnetic field falls with a one-over-distance law, thus localizing to its source, the charged particle. The existence of a conducting medium is an extreme case in that the electric and magnetic field are perfectly localized at their very source. However, the same is not true for the gravitational field, for it can propagate through the entire three-dimensional space: it cannot be shielded. For future convenience, one calls the electric charge to be a 0-*brane*; the electromagnetic field to be the *localized* field, and finally, the gravitational field to be the *bulk* field.

Now imagine a spacetime with $D{=}4{+}\delta$ dimensions, where δ is the total number of additional spatial dimensions. Consider a three-dimensional solitonic structure embedded in this D-dimensional spacetime such that all the SM particles, namely fermions, vector bosons and the Higgs field, are all stuck to this three-dimensional hypersurface. We name this particular three-dimensional surface as the 3-*brane*, and the entire D-dimensional space as *bulk*. In similarity with the above-mentioned example, gravity can propagate in the entire D-dimensional *bulk* whereas the SM fields can exist only in the 3-*brane*. The starting point of the analysis is the Einstein equations in D-dimensions [7, 8]:

$$R_{AB} - \frac{1}{2} R g_{AB} = -\frac{T_{AB}}{M_D^{2+\delta}}, \tag{1}$$

where $A = 0, 1, \ldots, 3, B = 1, 2, \ldots, \delta$. Here R_{AB} and R are, respectively, the Ricci tensor and the scalar curvature following from the repeated contractions of the Riemann tensor, T_{AB} is the energy momentum tensor of the matter, and M_D is the D-dimensional reduced Planck mass defined via $M_{pl} = \sqrt{V}_\delta M_D^{+\delta/2}, V_\delta$ being the volume of the δ-dimensional compactified space. The 3-*brane* on which we are stuck, in general, introduces a non-trivial metric background. However, it is expected that the surface tension of the brane cannot exceed the fundamental mass scale M_D, so that at distances $>> 1/M_D$ the metric will be essentially flat. In this case, as usual, one can expand g_{AB} around the flat D-dimensional metric η_{AB} via

$$g_{AB} = \eta_{AB} + \frac{2}{M_D^{1+\delta/2}} h_{AB}, \tag{2}$$

where $h_{AB}(x_\mu, \vec{y})$ is the 'perturbative' metric depending on four-dimensional spacetime coordinates x_μ as well as the additional spatial coordinates $\vec{y}$. Assuming a torus geometry T^δ for the extra dimensions, h_{AB} becomes periodic under $y_i \to y_i + 2\pi R$ for each i=1,2,..., δ. This periodicity condition entails the expansion [8]

$$h_{AB}(x_\mu, \vec{y}) = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \frac{1}{\sqrt{V_\delta}} h_{AB}^{(\vec{n})}(x_\mu) e^{i\frac{\vec{n}.\vec{y}}{R}}, \tag{3}$$

where $V_\delta = (2\pi R)^\delta$ is the volume of the torus T^6. Finally, since the entire matter content is situated on the 3-*brane*, one has necessarily $T_{AB}(x_\mu, \vec{y}) = \eta_A^\mu \eta_B^v T_{\mu v}$ with μ, v=0,. . .3. Then using the form of the metric (2) together with the expansion (3) in the Einstein equations, the D-dimensional metric tensor is seen to decompose into four types of canonically normalized Kaluza-Klein modes: $G_{\mu v}^{(\vec{n})}, V_{\mu i}^{(\vec{n})}, H^{(\vec{n})}$, and $S_{ij}^{(\vec{n})}$, where i, j=1,..., δ. These fields behave, in four-dimensional spacetime (time 3-*brane* itself), as a symmetric second rank tensor, vector scalar and scalar, respectively. However, as far as the SM particles are concerned, only two of these fields, $G_{\mu v}^{(\vec{n})}$ and $H^{(\vec{n})}$, are relevant as they are the only ones interacting with the matter. Besides, all of these four fields have identical masses: $m_n^2 \equiv \vec{n} \cdot \vec{n}/R^2$ for the $\vec{n} - th$ Kaluza-Klein level. The symmetric tensor field $G_{\mu v}^{(\vec{n})}$ has a total of five independent components after taking into account the constraints $\partial^\mu G_{\mu v}^{(\vec{n})}$=0 and ${G_\mu^{(\vec{n})}}^\mu$=0. Therefore, one can identify $G_{\mu v}^{(\vec{n})}$ with a spin-2 particle as 5=2s+1. It is tempting to call $G_{\mu v}^{(\vec{n})}$ as the *graviton*, and $H^{(\vec{n})}$ as *dilaton* Kaluza-Klein towers. In essence, a massless graviton in D=4+δ dimensions is seen to produce, among other fields, a massive graviton tower. In fact, this observation summarizes the modification in the Newton's force law: At a given distance R, two gravitating bodies attract each other via $1/R$ and $e^{-M_D R}/R$ type interactions for massless and massive gravitons, respectively. The interaction of graviton and dilaton towers with matter is represented by the interaction Lagrangian [8]:

$$L_{int} = -\frac{1}{M_{Pl}} [G_{\mu v}^{(\vec{n})} - \beta \eta_{\mu v} H^{(\vec{n})}] T^{\mu v}, \tag{4}$$

where $\beta^2 = (\delta - 1)/(3(\delta + 2))$. The model-dependent part of this interaction Lagrangian comes only from the energy-momentum tensor $T^{\mu v}$, which depends on the details of a given particle physics model.

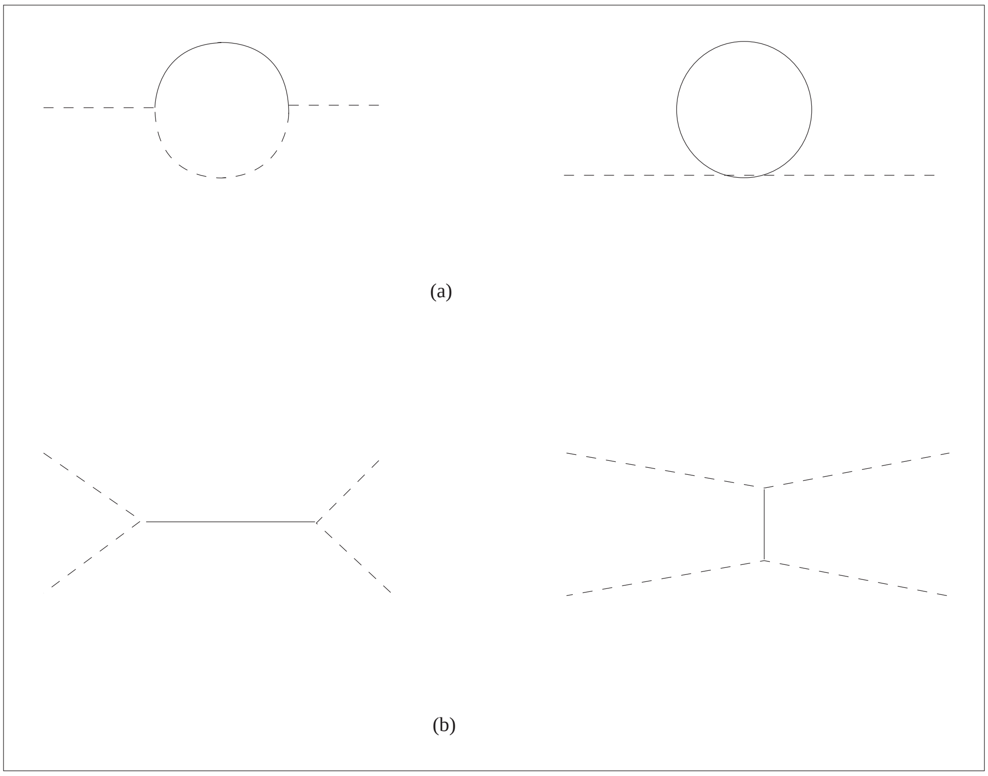

Figure 1. Relevant diagrams for computing δm_ϕ^2 (a) and $\delta\lambda$ (b) with dilaton and graviton contributions (solid line) that couple to the Higgs field ϕ (dashed line) via the interaction Lagrangian (4).

One crucial point to be noted about the interaction Lagrangian (4) is that it is Planck-suppressed. Thus one naively concludes that such an interaction is not important at all for scattering processes. However, one should notice that on infinite tower of Kaluza-Klein states for graviton and dilaton, when summed over which, causes orders of magnitude of enhancement in scattering processes. For future convenience, as well as to illustrate this enhancement effect, we show how to sum up an s-channel Kaluza-Klein exchange diagram:

$$\frac{1}{M_{Pl}^2}\sum_{\vec{n}}\frac{1}{s-m_n^2} \approx \frac{1}{M_{Pl}^2}\int\frac{(2\pi)^{\delta/2}}{\Gamma(\delta/2)} \mid \vec{n}\mid^{\delta-1} d\mid\vec{n}\mid \frac{1}{s-m_n^2}, \tag{5}$$

where the sum is approximated by an integral weighted by the density of Kaluza-Klein states. Using $m_n^2 \equiv \vec{n}\cdot\vec{n}/R^2$ and $M_{pl} = \sqrt{V_\delta} M_D^{+\delta/2}$, the above integral is converted to an integral over the Kaluza-Klein masses:

$$\frac{1}{M_{Pl}^2}\frac{1}{(2\pi)^{\delta/2}\Gamma(\delta/2)}\frac{M_{Pl}^2}{M_D^{2+\delta}}\int dm \frac{m^{\delta-1}}{s-m^2}. \tag{6}$$

The initial M_{Pl}^{-2} factor is cancelled, thanks to the summation over the Kaluza-Klein states which produces a factor M_{Pl}^{+2}. Therefore, despite the appearent Planck mass suppression in the interaction Lagrangian (4), summation over the Kaluza-Klein towers brings about a huge enhancement: $M_{Pl}/M_D \sim 10^{15}$ for $M_D \sim$ TeV.

The energy-momentum tensor of the SM Lagrangian is largely model-independent, except for the Yukawa and Higgs sectors due to lack of experimental information about the elements of the CKM matrix and the Higgs potential. The scalar sector of the SM Lagrangian is spanned by a single Higgs doublet [9].

$$\Phi = \begin{pmatrix} \phi + i\varphi z \\ \varphi_w^{\bar{\varphi}} \end{pmatrix}, \tag{7}$$

where φ_z and $\varphi_w^{\pm}$ are the Goldstone modes swallowed by Z and $W^{\pm}$ bosons to acquire their masses after electroweak symmetry breaking. The electroweak symmetry breaking, however, requires ϕ to acquire a non-vanishing vacuum expectation value in accordance with LEP and TEVATRON data. This non-vanishing vacuum expectation value for ϕ follows, for example, from a ϕ^4 potential,

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{\lambda}{4}\phi^4, \tag{8}$$

where $\lambda > 0$, for the potential to be bounded from below. Minimization of the potential gives

$$<\phi>\equiv v_0 = \begin{cases} (-m_\phi^2/\lambda)^{1/2}, \text{if } m_\phi^2 < 0 \\ 0, \text{if } m_\phi^2 > 0 \end{cases} \tag{9}$$

where the two cases correspond to *broken* and *symmetric* phases, respectively. Now the question comes: "What are the effects of the dilaton and graviton Kaluza-Klein towers on the Higgs potential given in (8)"? To answer this question one has to compute the effective mass $\hat{m}_\phi^2$ and the effective quartic coupling $\hat{\lambda}$, which require the evaluation of the diagrams (a) and (b) in Fig. 1, respectively. For the evaluation of these diagrams one needs the Feynman rules for Higgs-graviton and Higgs-dilaton couplings:

$$\phi\phi G_{\mu v}^{(\vec{n})} : -i\frac{\kappa}{2}\frac{m_\phi^2}{2}\eta_{\mu v} \tag{10}$$

$$\phi\phi H^{(\vec{n})} : -i2\kappa\frac{m_\phi^2}{2}\omega \tag{11}$$

$$\phi\phi G_{\mu v}^{(\vec{n})}G_{\rho\sigma}^{(\vec{n})} : -i\frac{\kappa^2}{4}\frac{m_\phi^2}{2}C_{\mu v\rho\sigma} \tag{12}$$

$$\phi\phi H^{(\vec{n})}H^{(\vec{n})} : -i4\kappa^2\frac{m_\phi^2}{2}\omega^2 \tag{13}$$

where $\kappa = 1/M_{Pl}, \omega^2 = 2/(3(\delta+2))$, and $C_{\mu v\rho\sigma} = \eta_{\mu\rho}\eta_{v\sigma} + \eta_{\mu\sigma}\eta_{v\rho} - \eta_{\mu v}\eta_{\rho\sigma}$. These Feynman rules have been written for a constant Higgs configuration; that is, Higgs fields at the external legs in Fig. 1 have vanishing momentum. This is motivated by the fact that the eventual concern is on the electroweak symmetry breaking, and the vacuum state has to have translational invariance. Then the parameters of the potential, after including the effects of the Kaluza-Klein towers, take the form

$$\begin{aligned} \hat{m}_\phi^2 &= \left\{1 + \frac{1}{\pi} f_m(x_\phi, \delta)\right\} m_\phi^2 \\ \hat{\lambda} &= \lambda + \pi f_\lambda(x_\phi, \delta) x_\phi^2, \end{aligned} \tag{14}$$

where $x_\phi = m_\phi^2/M_D^2$ after identifying the ultraviolet cut-off on the Kaluza-Klein summation with the D-dimensional reduced Planck mass M_D. The gravity-induced functions $f_{m,\lambda}(x_\phi, \delta)$ have the explicit expressions:

$$f_\lambda(x_\phi, \delta) = \frac{128}{3} \frac{\delta+1}{\delta^2 - 4} \tag{15}$$

$$\begin{aligned} f_m(x, \delta) = &- \frac{1}{3(\delta-2)} \left[5 + 6x - 6x^2 ln \frac{1+x}{x}\right] - \frac{4}{3\delta(\delta+2)} x \\ &\int_0^1 dy \left[\frac{32x + 30y + \delta(12x + 13y)}{y + x}\right] \mathcal{F}(1, \frac{\delta}{2}, 1 + \frac{\delta}{2}, -\frac{1}{y}), \end{aligned} \tag{16}$$

where $\mathcal{F}(a, b, c, x)$ is the Hypergeometric function of the second kind. The gravity-induced functions in (15) and (16) are valid only for $\delta \geq 3$. The interpretation of these results in the limiting case $M_D \to M_{Pl}$ certainly needs $\delta = 0$ due to the fundamental relation $M_{Pl} = \sqrt{V_\delta} M_D^{+\delta/2}$. Since the results presented above are no longer valid for $\delta = 0$, for M_D values close to M_{Pl}, the entire computation sholud be repeated. For the sake of clarity, it is convenient to list some highlights associated with these gravity-induced functions.

1. δm_ϕ^2 as well as $\delta\lambda$ are all proportional to m_ϕ^2. This is completely unusual compared to the usual gauge theories, where the radiative corrections are always proportional to the ultraviolet cut-off of the theory, M_D. Due to the gravitational nature of the interactions, the scalar mass m_ϕ^2, which is necessary for electroweak symmetry breaking, also breaks the conformal invariance of the theory. Indeed, the dilatation current j_D^μ is not conserved:

$$\partial_\mu j_D^\mu = T_\mu^\mu = m_\phi^2 \phi^2. \tag{17}$$

Since gravity couples to the energy momentum tensor $T_{\mu v}$, in the case of a translationally invariant vacuum state, the gravitational corrections are necessarily proportional to m_ϕ^2.

2. The term $x_\phi^2 log[1 + 1/x_\phi]$ in the expression of $f_m(x_\phi, \delta)$ requires special attention. This term vanishes when $x_\phi \to 0$. More interestingly, however, the algorithm here makes $f_m(x_\phi, \delta)$ *complex* when $m_\phi^2 < 0$ for $x_\phi < 0$ and $\mid x_\phi \mid < 1$. Since it is unlikely to have $\mid x_\phi \mid$ exceeding unity, it is guaranteed the *negative* $m_\phi^2 < 0$, which is necessary for spontaneous symmetry breaking in the absence of gravitational effects, forces $\hat{m}_\phi^2$ to be complex. This, in particular, makes the Higgs field *unstable* and induces a complex vacuum expectation value $\hat{m}_\phi^2/\hat{\lambda}$. One notes that this observation is independent of the total number of extra dimensions δ as well as particular values of x_ϕ; all that is required is $\mid x_\phi \mid < 1$. On the basis of these observations, especially due to the *instability* of the Higgs field, there is simply no possibility for $m_\phi^2 < 0$. In conclusion, only this logarithmic term is sufficient to rule out negative m_ϕ^2.

3. After the second item discussed above, there remains one last possibility: $m_\phi^2 > 0$ and again $\mid x_\phi \mid < 1$ for the consistency of the model. However, to decide if this case really breaks the electroweak symmetry, one has to analyze the gravity-induced function $f_m(x_\phi, \delta)$ in detail. Here the diffuculty follows from the fact that this function might have different signs in different portions of the parameter space.

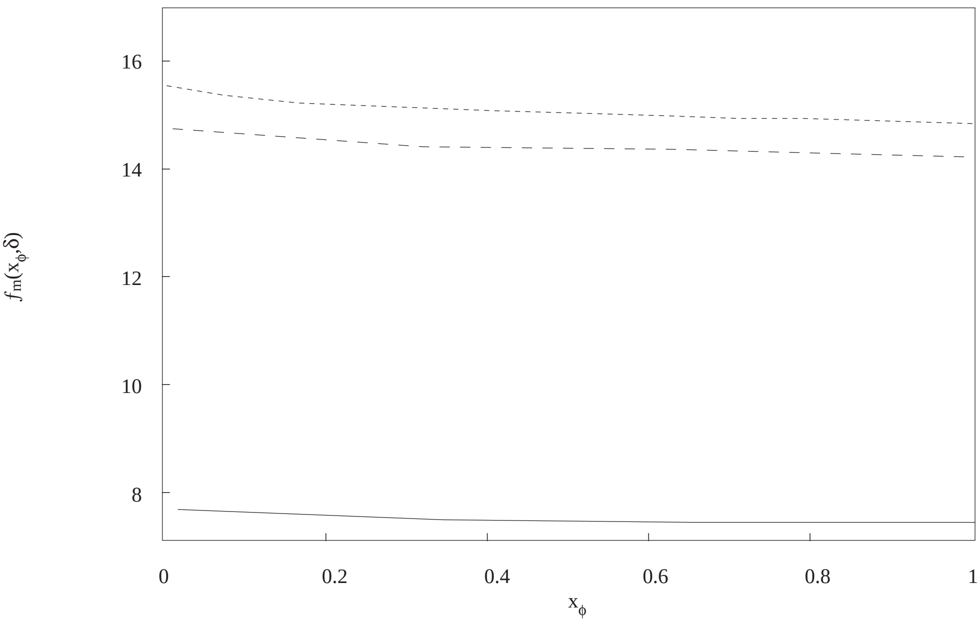

Figure 2. Variation of $f_m(x_\phi, \delta)$ with x_ϕ for δ=3 (solid curve), δ=10 (dashed curve) and δ=20 (dotted curve). The figure suggests that $f_m(x_\phi, \delta)$ remains positive for all dimensions and for all values of x_ϕ.

However, as is seen from the simple analysis in Fig. 2, $f_m(x_\phi, \delta)$ remains positive for all δ and x_ϕ. Thus, it is clear that this choice does not allow for electroweak symmetry breaking.

3. Discussions

In this work we have discussed the effects of low-scale quantum gravity models on the electroweak symmetry breaking. One can simply state that there is no allowance for electroweak symmetry breaking. One could relax this conclusion by allowing the Higgs field to be unstable in which case its mass parameter, and thus the vacuum expectation value, is complex. Since a complex vacuum expectation value implies CP-violation, this might further invoke new CP-violating contributions especially in Higgs interactions with the Kaluza-Klein towers. However, a more detailed analysis of the Higgs sector by taking into account those terms involving the momentum of the Higgs might change the conclusion to some extent. Such contributions could be important because they can introduce a renormalization of the Higgs field itself after making the identification: momentum^2=m_ϕ^2. Modulo such contributions, at the level of accuracy discussed in here, we conclude that low-scale quantum gravity prohibits the electroweak symmetry breaking, unless one allows for complex vacuum expectation value, or equivalently, extra CP-violation phases in the Yukawa sector.

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