

Dynamical relaxation of the CP phases in next-to-minimal supersymmetry

D. A. Demir

The Abdus Salam International Centre for Theoretical Physics, I-34100 Trieste, Italy
(February 7, 2008)

Abstract

After promoting the phases of the soft masses to dynamical fields corresponding to Goldstone bosons of spontaneously broken global symmetries in the supersymmetry breaking sector, the next-to-minimal supersymmetric model is found to solve the μ problem and the strong CP problem simultaneously with an invisible axion. The domain wall problem persists in the form of axionic domain formation. Relaxation dynamics of the physical CP-violating phases is determined only by the short-distance physics and their relaxation values are not necessarily close to the CP-conserving points. Having observable supersymmetric CP violation and avoiding the axionic domain walls both require nonminimal flavor structures.

PACS: 12.60.Jv, 11.30.Er, 12.60.Fr, 14.80.Mz

I. INTRODUCTION

The present bounds on the electric dipole moments (EDM) of the particles generate serious hierarchy problems concerning the amount of CP violation. To be specific, let us consider the neutron EDM [1]. It is well known that the QCD vacuum angle (θ_{QCD}) induces a neutron EDM which is approximately ten orders of magnitude larger than the existing bound. This is the source of the so-called strong CP problem, that is, $|\theta_{QCD}| \lesssim 10^{-10}$ instead of the expected order of unity. This naturalness problem is nicely solved by the celebrated Peccei–Quinn mechanism [2] which promotes θ_{QCD} to a dynamical variable via the phases of the quarks [2–4] or additional color triplets [5], and then relax it to zero thanks to the instanton-induced effective potential.

In the realm of supersymmetry in particular the minimal supersymmetric standard model (MSSM), there arises SUSY CP problem in addition to the strong CP problem. Indeed, the soft-breaking parameters as well as the μ parameter can have nonvanishing phases [6] leading to a neutron EDM exceeding the bounds by three orders of magnitude, except for certain portions of the SUSY parameter space where different sparticle contributions cancel [7]. However, it is clear that even if SUSY contributions partially cancel to agree with the experiment this is by no means a complete solution of the problem because the strong CP problem is still there.

A simultaneous solution to both strong CP and SUSY CP problems have been shown to exist [8] if there are spontaneously broken global symmetries in the SUSY-breaking sector. Here all soft masses as well as the μ parameter possess dynamical phases which realize the global symmetries in the SUSY-breaking sector nonlinearly. In effect, the relaxation mechanism proposed in [8] involves promoting the phases of the soft masses to dynamical variables so that (i) the phases in the quark and gluino mass matrices relax the QCD vacuum angle, and (ii) all phases appearing in the vacuum energy relax to CP-conserving points due to the radiative stability of the vacuum energy. While former solves the strong CP problem the latter does the SUSY CP problem. Therefore, the end result of this mechanism is that there is no source of CP violation beyond the Kobayashi–Maskawa (CKM) phase.

However, the MSSM suffers from a serious hierarchy problem: the μ puzzle. Namely, the Higgsino bilinear mass parameter, μ which follows from the superpotential, can be anywhere between the weak scale and the Planck scale [9]. In fact, the relaxation mechanism in [8] treats the μ parameter as a soft mass which is already stabilized at the weak scale. In this context, the next-to-minimal supersymmetric SM (NMSSM) [10] is the most economic extension of the MSSM in which the μ parameter is induced by the VEV of an additional gauge singlet. One notes that the NMSSM not only solves the μ problem but also offers a rich phenomenology for colliders [11] and dark matter [12].

In this work we will discuss dynamical relaxation of the CP phases in the NMSSM in order to check if it is possible to have a simultaneous solution to the hierarchy problems concerning the strong CP, SUSY CP and the μ parameter. It will be seen that the generalization of the μ parameter to a local operator modifies the infrared and ultraviolet sensitivities of the vacuum energy whereby offering a different relaxation mechanism. In particular, the radiative stability of the vacuum energy will no longer be sufficient to relax all CP-violating phases to CP-conserving points. In fact, there will be remnant physical phases that can contribute to CP-violating quantities like EDMs or neutral meson mixings.

In Sec. II we identify the possible sources of explicit CP violation in the tree level NMSSM lagrangian. We then determine the symmetries of the superpotential, and thereby show the need for promoting the phases to dynamical variables. Here we also list down all possible phase-dependent operators which can contribute to the vacuum energy.

In Sec. III we analyze the dynamical relaxation processes for the phases. First we discuss the relaxation of the QCD vacuum angle. Next we estimate radiative corrections to the vacuum energy, and show that the SUSY CP-violating phases no longer relax to the CP-conserving points.

In Sec. IV we discuss the MSSM limit and show that the differences between the two models follow mainly from their global symmetries and assumptions about the μ parameter.

In Sec. V we summarize the main results of the work and compare them with those of the MSSM in a tabular manner. We discuss also the implications of the results for other CP-violation phenomena.

II. CP-VIOLATING PHASES IN THE NMSSM

The superpotential of the NMSSM is given by

$$\hat{W} = h_s \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3} k_s \hat{S}^3 + h_u \hat{Q} \cdot \hat{H}_u \hat{U}^c + h_d \hat{Q} \cdot \hat{H}_d \hat{D}^c + h_e \hat{L} \cdot \hat{H}_d \hat{E}^c \quad (1)$$

where the Yukawa couplings h_s, \dots, h_e are non-hermitian matrices in the flavor space. The lagrangian of the model consists of several complex parameters

$$-\mathcal{L} \ni \left(h_s k_s^* S^{*2} H_u \cdot H_d + \text{H.c.} \right) + \left(\frac{1}{2} m_\lambda \tilde{\lambda} \tilde{\lambda} + A_s S H_u \cdot H_d + \frac{1}{3} A_k S^3 \right. \\ \left. + A_u Q \cdot H_u U^c + A_d Q \cdot H_d D^c + A_e L \cdot H_d E^c + \text{H.c.} \right) \quad (2)$$

where the first term is the F -term contribution, and the rest are all soft SUSY-breaking ones. In general the gaugino masses m_λ as well as the triscalar couplings $A_{s,\dots,e}$ are all complex quantities. The Higgs sector of the theory has three parameters, $h_s k_s^*$, A_s and A_k which can violate CP. After all phase redefinitions of the Higgs fields there remains one independent phase, say $\text{Arg}\{h_s k_s^*\}$, which violates CP explicitly. This can be seen from the fact that the mass-eigenstate scalars are mixtures of different CP eigenstates [13]. This tree level CP violation is a property of the NMSSM. For comparison, one notes that in the MSSM there is no such effect because possible phase of m_{12}^2 appearing in $m_{12}^2 H_u \cdot H_d$ can be rotated away after a phase redefinition of the Higgs doublets. Therefore, the only way for generating CP violation in the MSSM comes by the radiative corrections [6].

The NMSSM superpotential (1) possesses a global continuous R -symmetry, $\mathcal{U}(1)_R$ ¹. In fact, this would be a symmetry of the whole lagrangian were it not for the finite gaugino masses (m_λ) and A -terms. Therefore, the soft-breaking terms break the $\mathcal{U}(1)_R$ symmetry down to its Z_3 subgroup which causes the formation of domain walls [14]. With finite soft masses the only way of keeping $\mathcal{U}(1)_R$ symmetry unbroken is to let them have dynamical phases transforming as

$$(m_\lambda, A_s, A_k, A_u, A_d, A_e) \rightarrow e^{-iR_W \alpha} (m_\lambda, A_s, A_k, A_u, A_d, A_e) \quad (3)$$

where α is the $\mathcal{U}(1)_R$ rotation angle, and R_W is the charge of the superpotential (1) [15]. Therefore all of the gaugino masses and triscalar couplings are now spurions with charges compensating those of the fields, and the theory has a unique global symmetry $\mathcal{U}(1)_R$. The same operation of promoting the soft phases to dynamical variables in the MSSM, however, results in two global symmetries: an R -symmetry and a Peccei-Quinn symmetry [8].

Let us first discuss the tree level vacuum energy. After introducing the dynamical phases (3) the NMSSM vacuum manifold is described by the Higgs fields, S , H_u and H_d , and dynamical phases of A_s and A_k . Therefore, a direct minimization of the vacuum energy gives the relations

$$\text{Arg}[S H_u \cdot H_d] = \text{Arg}[A_s^*], \quad \text{Arg}[S^3] = \text{Arg}[A_k^*], \quad \text{Arg}[A_s A_k^*] = \theta_s - \theta_k, \quad (4)$$

¹ Under an R -rotation θ variable has charge $R_\theta = R_\lambda = R_W/2$. If a chiral superfield has charge R_χ then the charges of its scalar and fermionic components are, respectively, R_χ and $R_\chi - R_\theta$.

where $\theta_s = \text{Arg}[h_s]$, $\theta_k = \text{Arg}[k_s]$, and the Higgs fields here are replaced by their vacuum expectation values (VEV). These relations fix the phases of the Higgs fields in terms of A_s and A_k phases such that the relative phase between A_s and A_k equals the phase content of the F -term contribution in (2). Similar relations are also found in the MSSM: $\text{Arg}[m_{12}^2] = -\text{Arg}[H_u \cdot H_d]$ after minimizing the vacuum energy [8,6].

Any physical quantity with a nontrivial phase content is restricted to depend only $\mathcal{U}(1)_R$ -invariant combinations of the mass parameters. Since the Higgs fields eventually acquire VEVs they can also be included in the list of phase-dependent invariants. Depicted in Table I are sets of phase-dependent invariants having mass dimension $d = 2$ (first column) and $d = 4$ (second column). The phase of each combination is listed in the third column in terms of the independent phases

$$\phi_s(x) = \text{Arg}[m_\lambda A_s^*], \quad \phi_k(x) = \text{Arg}[m_\lambda A_k^*], \quad \phi_f(x) = \text{Arg}[m_\lambda A_f^*], \quad (5)$$

which will prove useful in comparing the results with those of the MSSM. In Table I, $|H|^2 = \{|S|^2, |H_u|^2, |H_d|^2\}$ stands for the quadratics of the Higgs fields. These phase-dependent operators are all determined by the R -invariance arguments. However, their contribution to the vacuum energy depends on the diagrammatics, and this will be done in the next section. The radiative stability of the vacuum with respect to the phases will realize the relaxation process.

Non-marginal operators (d = 2)	Marginal operators (d = 4)	phase content
$A_s A_k^*$	$A_s S^3, A_s A_k^* H ^2$	$\phi_s - \phi_k$
$A_f A_s^*$	$A_f S H_u \cdot H_d, A_f A_s^* H ^2$	$\phi_f - \phi_s$
$A_f A_k^*$	$A_f S^3, A_f A_k^* H ^2$	$\phi_f - \phi_k$
$m_\lambda A_s^*$	$m_\lambda S H_u \cdot H_d, m_\lambda A_s^* H ^2$	ϕ_s
$m_\lambda A_k^*$	$m_\lambda S^3, m_\lambda A_k^* H ^2$	ϕ_k
$m_\lambda A_f^*$	$m_\lambda A_f^* H ^2$	ϕ_f
	$m_\lambda A_f A_k^* A_s^*$	$\phi_k + \phi_s - \phi_f$
	$m_\lambda A_f^* A_k A_s^*$	$\phi_f + \phi_s - \phi_k$
	$m_\lambda A_f^* A_k^* A_s$	$\phi_k + \phi_f - \phi_s$

TABLE I. Phase-dependent, R -invariant, $d = 2$ and $d = 4$ operators and their phases.

III. DYNAMICAL RELAXATION OF THE CP PHASES

It is clear from the superpotential (1) that, the fermion superfields cannot be assigned R -charges like $R_Q = -R_U$ or $R_Q = -R_D$ or $R_L = -R_E$; therefore, $\mathcal{U}(1)_R$ has to have a quantum mechanical anomaly with respect to both QCD and QED. Then $\mathcal{U}(1)_R$ is a nonlinearly-realized global symmetry of the lagrangian, and the phase fields above are nothing but the Goldstone bosons of some spontaneously broken global symmetries in the SUSY-breaking

sector. The QCD anomaly of $\mathcal{U}(1)_R$ shifts the QCD vacuum angle as $\theta_{QCD} \longrightarrow \theta_{QCD} + \mathcal{G}_R(x)$. Since $\mathcal{G}_R(x)$ is a Goldstone boson it would have a strictly flat potential were not it for the instanton effects in the QCD vacuum which develops a potential such that $\langle \mathcal{G}_R(x) \rangle = -\theta_{QCD}$ whereby solving the strong CP problem [2,3,5,4]. The resulting massive pseudoscalar is the R -axion, a_R , having mass $m_R \sim m_\pi f_\pi / M_{SUSY}$ and decay constant $f_R \sim M_{SUSY}$. These axion parameters are in the invisible *axion window* [16] as long as M_{SUSY} refers to an intermediate scale. In fact, the R -axion here is both a KSVZ axion (through the gluino phase) and DFSZ axion (through the phases of the Higgs doublets). Needless to say, exactly the same kind of relaxation effect occurs also in the MSSM with the associated R -symmetry [8].

The soft breaking lagrangian (2) consists of four independent phases $\text{Arg}[A_s]$, $\text{Arg}[A_k]$, $\text{Arg}[A_f]$ and $\text{Arg}[m_\lambda]$. Clearly one combination of these four fundamental phases is spent in relaxing the QCD vacuum angle. However, the remaining three phases $\phi_s(x)$, $\phi_k(x)$ and $\phi_f(x)$ cannot be determined through the QCD effects. Therefore, one must check the long-distance (electroweak interactions) and short-distance (interactions around Planck scale) contributions to find their relaxation points.

Relaxation of the effective QCD vacuum angle occurs via its instanton-induced potential. Similar to this, we now look for the possibility of inducing potentials for the phases of the operators in Table I. To do this we compute the coefficients of these operators. The operators with mass dimension four (the second column in Table I) are *marginal* operators, that is, their contribution to the vacuum energy is always weighted by dimensionless coefficients. On the other hand, operators with dimension two (first column of Table I) are *non-marginal* operators in the sense that their coefficients should have always mass dimension two. The divergences of the *marginal* operators could be at most *logarithmic* whereas those of the *non-marginal* ones are *quadratic*.

We first calculate the contributions of the *marginal* operators to the vacuum energy. Depicted in Fig. 1 is a set of sample diagrams that generate some of the operators in the second column of Table I. Here (a), (b), (c) and (d) generate, respectively, $h_s h_f^* A_f S H_u \cdot H_d$, $h_s m_\lambda S H_u \cdot H_d$, $h_s m_\lambda A_s^* |S|^2$ and $h_f m_\lambda A_f^* |H_u|^2$. All the remaining dimension-four operators can be generated using similar diagrams. For example, the diagram (e) in Fig. 1 generates $h_t k_s h_s^* m_\lambda A_f^* A_k^* A_s$. One notices that evaluation of each diagram in Fig. 1 produces a *marginal* operator belonging to Table I; however, each contribution is conveyed by a non-dynamical phase associated with the Yukawa couplings. Letting the loop momenta vary from $m_{3/2}$ to M_{Pl} , on dimensional grounds, the *marginal* operators contribute to the vacuum energy as

$$\begin{aligned}
(\Delta V)_{long} = m_{3/2}^4 \log \left(\frac{M_{Pl}^2}{m_{3/2}^2} \right) & \left\{ c_a \cos(\phi_s - \phi_f + \theta_s - \theta_f) + c_{b,c} \cos(\phi_s + \theta_s) \right. \\
& \left. + c_d \cos(\phi_f + \theta_f) + c_e \cos(\phi_f + \phi_k - \phi_s + \theta_f + \theta_k - \theta_s) + \dots \right\} \quad (6)
\end{aligned}$$

where the subscript *long* emphasizes that this potential is sensitive to long-distance ($m_{3/2}^{-1}$) physics; its dependence on short-distance (M_{Pl}^{-1}) physics is only logarithmic. In this expression the dimensionless parameters c_a, \dots, c_e are, respectively, the weights of the diagrams

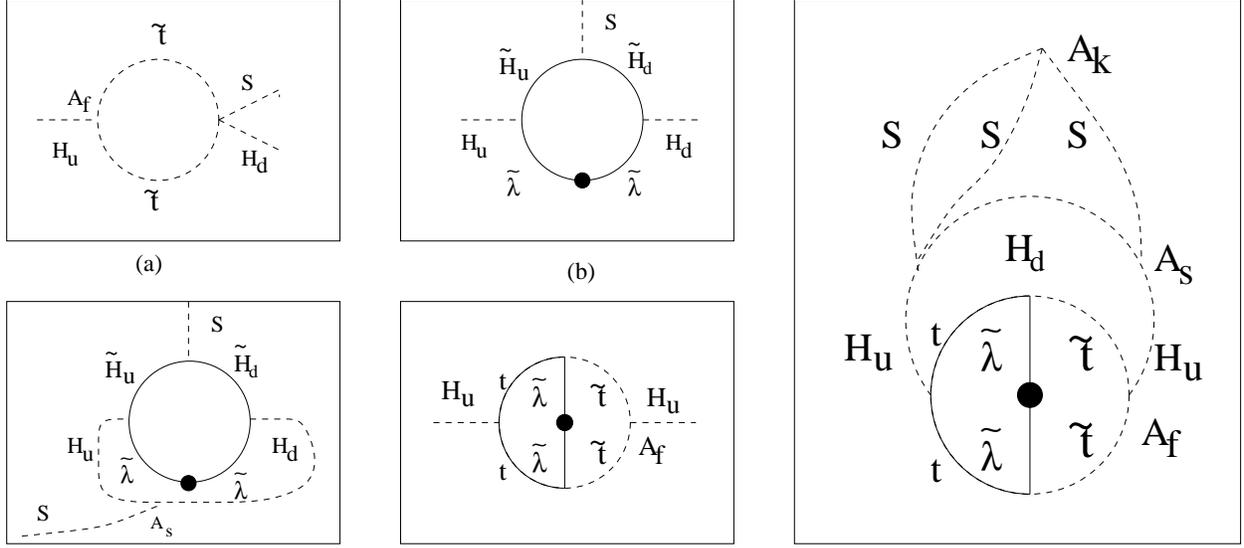


FIG. 1. Sample diagrams generating some of the *marginal* operators in Table I. The dot on the gaugino line shows the gaugino mass insertion.

(a), \dots , (e) in Fig. 1, and the ellipses stands for contributions of the diagrams that generate other marginal operators listed in Table I. Here the weight factors c_i consist of the Yukawa and gauge couplings as well as the loop suppression factors.

After estimating the contributions of *marginal* operators, we now compute those of the *non-marginal* operators in the first column of Table I. Depicted in Fig. 2 are the loop diagrams ((a), \dots , (f)) generating the relevant operators in rows (1), \dots , (6) of Table I, respectively. On dimensional grounds, the *non-marginal* operators contribute to the vacuum energy as follows

$$\begin{aligned}
(\Delta V)_{short} = & \frac{M_{Pl}^2 m_{3/2}^2}{(4\pi)^6} \left(\overline{c}_f \cos(\phi_f + \theta_f + \overline{\delta}_f) + \overline{c}_s \cos(\phi_s + \theta_s + \overline{\delta}_s) \right. \\
& + \overline{c}_{sk} \cos(\phi_k - \phi_s + \theta_k - \theta_s + \overline{\delta}_{sk}) + \overline{c}_{fs} \cos(\phi_s - \phi_f + \theta_s - \theta_f + \overline{\delta}_{fs}) \\
& \left. + \frac{\overline{c}_{fk}}{(4\pi)^2} \cos(\phi_k - \phi_f + \theta_k - \theta_f + \overline{\delta}_{fk}) + \frac{\overline{c}_k}{(4\pi)^2} \cos(\phi_k + \theta_k + \overline{\delta}_k) \right) \quad (7)
\end{aligned}$$

where the subscript *short* stresses that this contribution is highly sensitive to short-distance physics due to its quadratic dependence on M_{Pl} . In this expression, \overline{c}_i stands for the weight of the i -th diagram in Fig. 2, and $\overline{\delta}_i$ its possible phase shift beyond the ones coming from the Yukawa couplings. One notices that \overline{c}_i here does not include the loop factors; they are functions of only Yukawa and gauge couplings. The additional phase shifts $\overline{\delta}_i$ represent possible sources of CP-violation at short-distances. If CP is spontaneously broken together with supersymmetry they can be taken at the CP conserving points. This is the case is in the supergravity scenarios [17] so that we will take it granted by setting all $\overline{\delta}_i$ to zero [8].

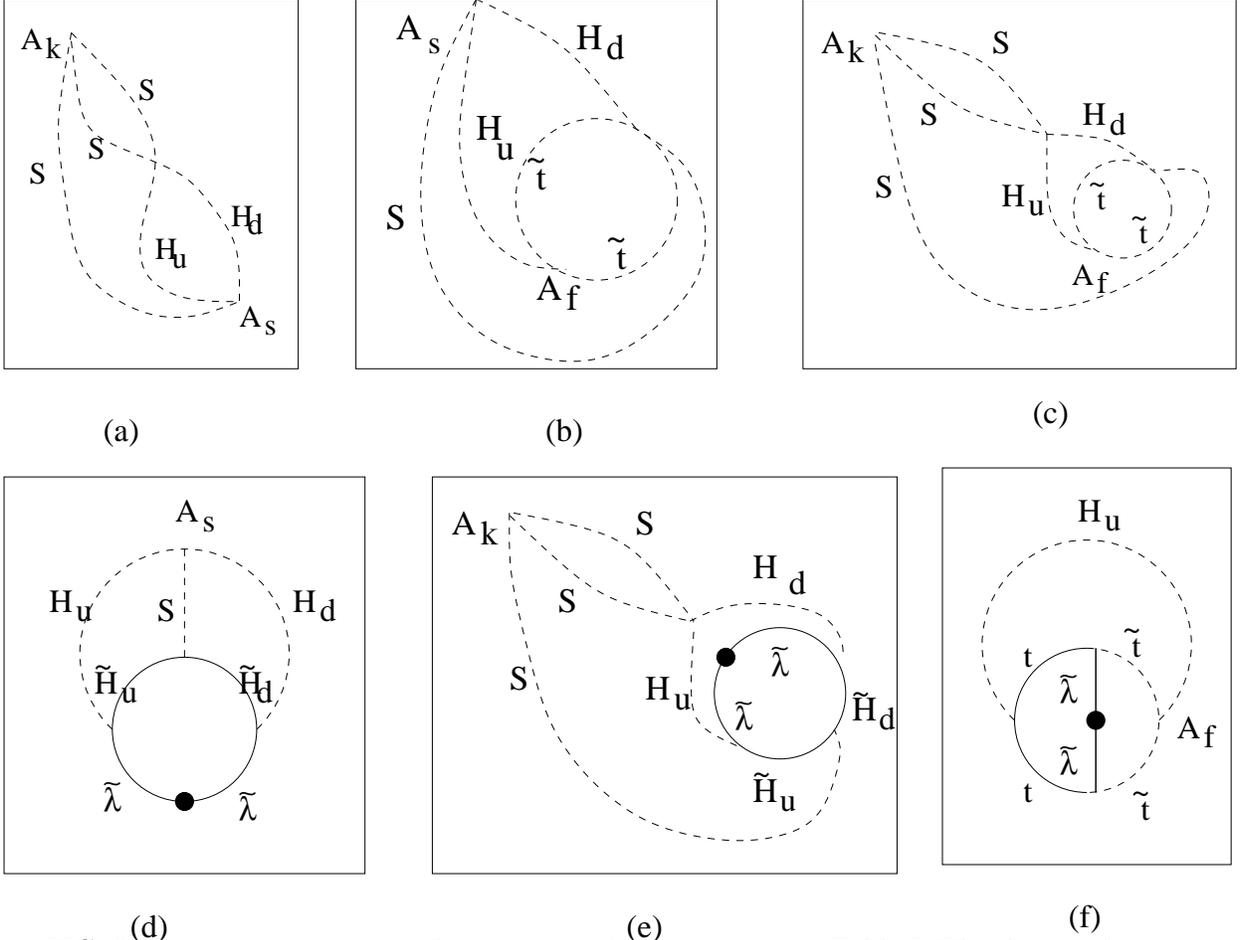


FIG. 2. Diagrams generating the *non-marginal* operators in Table I. The dot on the gaugino line represents the gaugino mass insertion.

So far we have assumed all Yukawa couplings to have arbitrary nondynamical phases. However, it is possible to rotate the quark fields such that the phases of h_u and h_d are transferred to charged current vertices via the CKM matrix. In the SM the CKM matrix is the only source of CP violation, and its size is characterized by the Jarlskog invariant $J = \text{Im}[V_{ud}V_{td}^*V_{td}V_{ub}^*]$. Contribution of this parameter to the vacuum energy is of $\mathcal{O}(\alpha/4\pi)^2 J$ which is much smaller than the supersymmetric ones. Indeed, the Higgs sector Yukawa couplings (h_s and h_k) have important differences from h_u and h_d concerning the loop structures in Figs. 1 and 2.

The vacuum energy with radiative corrections takes the form

$$V(\phi_f, \phi_s, \phi_k) = V_{tree} + (\Delta V)_{short}(\phi_f, \phi_s, \phi_k) + (\Delta V)_{long}(\phi_f, \phi_s, \phi_k) \quad (8)$$

which has to be minimized with respect to $\phi_k(x)$, $\phi_f(x)$ and $\phi_s(x)$. It is worth noting that there is no particular operator in Table I which has only long-distance sensitivity, that is, all of the dynamical phases appear in both (6) and (7). One further notes that V_{short} has

a large weight factor due to its quadratic M_{Pl} dependence. Therefore, all extremization equations for (8) are saturated by the short–distance contribution,

$$\langle \phi_f \rangle = \langle \phi_k \rangle + \theta_k = \langle \phi_s \rangle + \theta_s + \pi \quad (9)$$

after assuming that all \overline{c}_i are of similar order of magnitude, which is reasonable. It is worthy of noting that none of the phases develops a VEV in close proximity of a CP–conserving point: $\langle \phi_f \rangle, \langle \phi_k \rangle, \langle \phi_s \rangle \neq 0, \pi$. It is mainly here that there is an important difference between the NMSSM and the MSSM: The latter has all phases relaxing the CP–conserving points whereas the former does not. In the next section we will discuss the reason for this difference by considering the MSSM limit of the NMSSM.

The physical Goldstone bosons $\mathcal{G}_{f,k,s}(x) \equiv M_{SUSY} (\phi_{f,k,s}(x) - \langle \phi_{f,k,s} \rangle)$, are massive pseudo–scalars with masses

$$m_{k,s,f}^2 \sim M_{Pl}^2 m_{3/2}^2 / M_{SUSY}^2. \quad (10)$$

Therefore, particular short–distance sensitivity of the potentials of $\phi_{f,k,s}(x)$ require the pseudo–Goldstone bosons $\mathcal{G}_{f,k,s}(x)$ to have masses right at the intermediate scale. These pseudo–Goldstone bosons have only derivative couplings to the visible matter so that they are invisible to collider experiments.

During the entire analysis the triscalar couplings in (2) are written without Yukawa couplings, for convention. If required, one can separate the Yukawa couplings by the replacement, $A_s \rightarrow h_s A_s$, $A_k \rightarrow k_s A_k$ and $A_f \rightarrow h_f A_f$. Then the VEV’s in (9) give

$$\langle \text{Arg}[m_\lambda A_f^*] \rangle = \langle \text{Arg}[m_\lambda A_k^*] \rangle = \langle \text{Arg}[m_\lambda A_s^*] \rangle + \pi, \quad (11)$$

which are independent of the Yukawa phases. This replacement separates dynamical and nondynamical phases, and still VEV’s of the phases do not relax to a CP–conserving points. From this relation it follows that if any of these phases relaxes to a CP–conserving point by some reason so do the remaining two.

IV. THE MSSM LIMIT

To clarify the meaning of the CP–violating relaxation points in (9) or (11) it may be convenient to discuss the MSSM limit both algebraically and diagrammatically. The main difference between the MSSM and NMSSM follows from their symmetries and structures of the superpotentials. With purely triscalar nature of the soft terms in (2), the scalar potentials of all pseudo–Goldstone bosons turn out to be controlled by the short–distance physics, in particular, there is no phase–dependent invariant that receives a potential only from the long–distance physics. This is not the case in the MSSM as one of the phases receives a potential only from the long–distance effects [8] so that its relaxation dynamics is different than those of the remaining two. In the conventions leading to (11), the MSSM limit is realized by the replacements

$$h_s S \rightarrow \mu, \quad h_s A_s S \rightarrow m_{12}^2, \quad k_s \rightarrow 0, \quad (12)$$

where now μ and m_{12}^2 are no longer dynamical fields, instead they are background spurions that can appear only as mass insertions in the loop diagrams. It may be convenient to discuss the modifications in the topologies of the diagrams in Fig. 2 under the replacements (12). It is clear that diagrams (a), (c) and (e) vanish due to vanishing k_s . However, as Fig. 3 shows explicitly, the three-loop diagrams (b) and (d) in Fig. 2 go over to two-loop diagrams (\bar{b}) , (\bar{d}) whereas the diagram (f) remains unaffected. A simple observation on Fig. 3 shows that (\bar{b}) , (\bar{d}) and (\bar{f}) generate, respectively, the phases $\text{Arg}[\mu A_f m_{12}^{*2}] \equiv \phi_s - \phi_f$, $\text{Arg}[m_\lambda \mu m_{12}^{*2}] \equiv \phi_s$ and $\text{Arg}[m_\lambda A_f] \equiv \phi_f$. It should be emphasized that these phases are precisely the ones appearing in the MSSM [8].

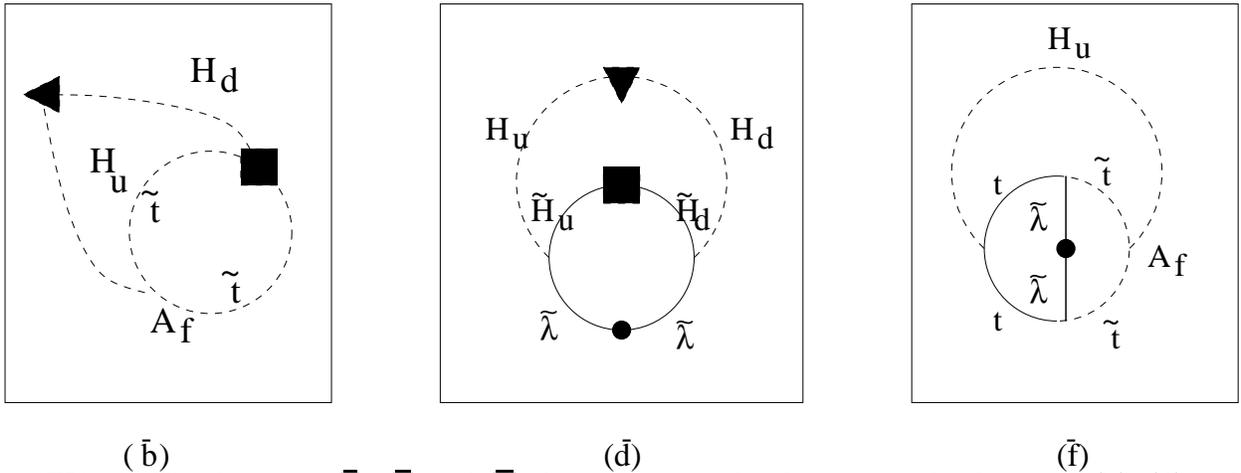


FIG. 3. The diagrams (\bar{b}) , (\bar{d}) and (\bar{f}) show, respectively, the form of the diagrams (b), (d) and (f) of Fig. 2 in the MSSM limit. Here the dot, triangle and square correspond to the insertions of the mass parameters, m_λ , m_{12}^2 and μ , respectively.

As is clear from the modifications in the topologies of the diagrams in Fig. 2 and Fig. 3, types of the divergences of the diagrams are modified. Indeed, unlike (b) and (d) of Fig. 2, now (\bar{b}) and (\bar{d}) are only *logarithmically* divergent. This is, in fact, what is implied by the spurion character of the singlet field in the MSSM limit. It is this effect that generates the *marginal* operators $\mu A_f m_{12}^{*2}$ and $m_\lambda \mu m_{12}^{*2}$ which are only logarithmically divergent. On the other hand, the MSSM limit does not alter the logarithmic structure of (6) apart from certain modifications in individual diagrams. Therefore, summing up the contributions of all diagrams, the radiative corrections to the vacuum energy takes the form

$$\begin{aligned}
 (\Delta V)_{\text{MSSM}} = & m_{3/2}^4 \log \left(\frac{M_{Pl}^2}{m_{3/2}^2} \right) \left[c_s \cos(\phi_s + \delta_s) + c_{sf} \cos(\phi_s - \phi_f + \delta_{sf}) \right] \\
 & + m_{3/2}^2 M_{Pl}^2 c_f \cos(\phi_f + \delta_f) .
 \end{aligned} \tag{13}$$

In this expression the weight factors and the phase shifts have the same meaning as in (6) and (7), and the loop suppression factors are not factored out. Taking again the phase shifts at CP-conserving points in both short- and long-distance contributions, one finds that

$\langle\phi_f\rangle = 0$ or π to an accuracy $\mathcal{O}(m_{3/2}^2/M_{Pl}^2)$. Then, necessarily $\langle\phi_s\rangle = 0$ or π . Therefore, $\langle\phi_f\rangle$ ($\langle\phi_s\rangle$) is determined solely by the short–distance (long–distance) dynamics. Moreover, both phases relax to CP–conserving points. As a result, in the MSSM limit (12) the phases in (9) and (11) go over the usual MSSM relaxation pattern leaving ϕ_k completely undetermined.

It is, in fact, the $k_s \rightarrow 0$ limit that washes out any information about the fate of ϕ_k . Obviously, for $k_s \equiv 0$, the NMSSM superpotential (1) possesses an additional $U(1)_{PQ}$ symmetry, and therefore, $\langle\phi_k\rangle$ is nothing but the phase lifted by this global symmetry. Therefore, relaxation of the physical CP violating phases away from the CP–conserving points in (9) or (11) follows from the fact that this $U(1)_{PQ}$ symmetry is broken by a dimensionless parameter k_s carrying no information about the SUSY breaking sector.

V. CONCLUSIONS AND DISCUSSIONS

In Table II we list the main results of the work in comparison with the MSSM predictions [8]. Here the upper block refers to the non–dynamical phases whereas the lower block is for the same models with dynamical phases. As the table shows, all three NMSSM pseudo–Goldstone bosons have masses around M_{SUSY} so that they are too heavy to appear in the existing colliders. On the other hand in the MSSM one of the pseudo–Goldstone bosons fall below the TeV–scale.

	MSSM	NMSSM
μ – problem	yes	no
domain walls	no	yes
strong CP problem	yes	yes
sources for SUSY CP violation	yes	yes
global symmetries		\mathcal{Z}_3

μ – problem	yes	no
axionic domain walls	yes	yes
strong CP problem	no	no
sources for SUSY CP violation	no	yes
# of pseudo–Goldstone bosons	2	3
and their masses	$\sim m_{3/2}^2/M_{SUSY}$ and $\sim M_{SUSY}$	all have $\sim M_{SUSY}$
global symmetries	$U(1)_{PQ} \times U(1)_R$	$\mathcal{U}(1)_R$

TABLE II. MSSM vs NMSSM without (upper block) and with (lower block) dynamical phases. Unlike the MSSM, the NMSSM provides sources for SUSY CP violation, solves the μ problem, and has all its physical Goldstone bosons with masses $\mathcal{O}(M_{SUSY})$. Moreover, both models solve the strong CP problem at the expense of developing axionic domain walls.

Having finite SUSY CP violation with vanishing QCD vacuum angle is an important property of NMSSM not found in the MSSM. Indeed, with finite θ_{QCD} , it does not matter

if one cancels the SUSY contributions to EDMs [7], because the QCD contribution anyhow exceeds the bound by several orders of magnitude. Therefore, it is in the NMSSM that one can safely take SUSY and CKM phases as the mere sources of CP violation, that is, EDMs of the electron and neutron as well as ϵ_K and ϵ'_K/ϵ_K can all be calculated without worrying about the effects of the nonperturbative strong interactions. It is a question of the SUSY parameter space if the calculated values for these observables agree with the experiment.

It may be useful to recall here the relevance of the flavor structure. Given the CLEO determination of $\text{BR}(B \rightarrow K^*\gamma)$ then there is no possibility that the SUSY CP violation can saturate the observed CP violation with minimal flavor structure [18]. Though having finite SUSY CP violation is necessary it is by no means sufficient; one has to look for flavor structures beyond the usual Yukawa hierarchies.

It is a rather general statement [19] that all Peccei–Quinn type solutions to the strong CP problem suffer from the axionic domain walls, which are disastrous cosmologically [14,20]. Although axionic wall formation is an additional problem for the MSSM, it exists in the NMSSM with and without the dynamical phases so that phase relaxation does not generate a new difficulty in this model. One of the ways of avoiding the axionic wall formation is to embed the R –symmetry into the center of a GUT group [21]. However, in non–GUT gauge structures like MSSM and NMSSM this method does not work. The only remaining way out of this difficulty is to choose nonminimal flavour structures [22] such that domain wall number is reduced to unity. It is in this sense that the question of having observable SUSY CP violation and avoiding the domain walls might be answered by a common flavor structure. One further notes that the quadratic short–distance sensitivity of the NMSSM vacuum energy arises also in the dynamical flavor matrices [23], Yukawa couplings [24] and determinations of the SUSY–breaking scale [25].

VI. ACKNOWLEDGEMENTS

The author is grateful to Goran Senjanović for numerous discussions and helpful comments about various aspects of this work. The author thanks to Charanjit Aulakh and Antonio Masiero for useful discussions and their careful reading of the manuscript.

REFERENCES

- [1] P. G. Harris *et. al.*, Phys. Rev. Lett. **82**, 904 (1999).
- [2] R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
- [3] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
- [4] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **B104**, 199 (1981); A. Zhitnitskii, Sov. J. Nucl. Phys. **31**, 260 (1980).
- [5] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B166**, 493 (1980).
- [6] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. B **255** (1985) 413; D. A. Demir, Phys. Rev. **D60** (1999) 055006, Phys. Rev. **D60** (1999) 095007, Phys. Lett. **B465** (1999) 177, hep-ph/9907279; A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. **B553** (1999) 3.
- [7] T. Ibrahim and P. Nath, Phys. Rev. **D57**, 478 (1998); Phys. Lett. **B418**, 98 (1998); Phys. Rev. **D58**, 111301 (1998); hep-ph/9910553; M. Brhlik, G. J. Good, and G. L. Kane, Phys. Rev. **D59**, 115004 (1999).
- [8] S. Dimopoulos and S. Thomas, Nucl. Phys. **B465** (1996) 23.
- [9] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. **D27** (1983) 2359; J.E. Kim and H.P. Nilles, Phys. Lett. **B138** (1984) 150; K. Inoue, A. Kakuto, and T. Takano, Prog. Theor. Phys. **75** (1986) 664; A. A. Anselmi and A. A. Johansen, Phys. Lett. **B200** (1988) 331.
- [10] P. Fayet, Nucl. Phys. **B90** (1975) 104; H.-P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. **B120** (1983) 346; J.-M. Frere, D. R. T. Jones, and S. Raby, Nucl. Phys. **B222** (1983) 11; J.-P. Derendinger and C. A. Savoy, Nucl. Phys. **B237** (1984) 307; L. Durand and J. L. Lopez, Phys. Lett. **B217** (1989) 463; M. Drees, Intern. J. Mod. Phys. **A4** (1989) 3645; J. Ellis, J. Gunion, H. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. **D39** (1989) 844.
- [11] D. A. Demir, hep-ph/9902468; J. Espinosa and M. Quiros, Phys. Lett. **B279** (1992) 92, **B302** (1993) 51; U. Ellwanger and M. Lindner, Phys. Lett. **B301** (1993) 365; U. Ellwanger and M. Rausch de Traubenberg, Z Phys **C53** (1992) 521; U. Ellwanger, Phys. Lett. **B303** (1993) 271; G. Kane, C. Kolda, and J. Wells, Phys. Rev. Lett. **70** (1993) 2686; T. Elliott, S.F. King, and P.L. White, Phys. Lett. **B305** (1993) 71, **B314** (1993) 56; P.N. Pandita, Phys. Lett. **B318** (1993) 338; Z Phys **C59** (1993) 575; T. Elliott, S.F. King, and P.L. White, Phys. Rev. **D49** (1994) 2435; U. Ellwanger, M. Rausch de Traubenberg, and C.A. Savoy, Z.Phys. **C67** (1995) 665; F. Franke and H. Fraas, Phys. Lett. **B336** (1994) 415.
- [12] B.R. Greene and P.J. Miron, Phys. Lett. **B168** (1986) 226; R.A. Flores, K. Olive and D. Thomas, Phys. Lett. **B263** (1991) 425; K. Olive and D. Thomas, Nucl. Phys. **B335** (1991) 192; S.A. Abel, S. Sarkar, and I.B. Whittingham, Nucl. Phys. **B392** (1993) 83.
- [13] N. Haba, M. Matsuda, and M. Tanimoto, Prog. Theor. Phys. Suppl. **123** (1996) 265.
- [14] C. Panagiotakopoulos and K. Tamvakis, Phys. Lett. **B446** (1999) 224; S.A. Abel, S. Sarkar, and P.L. White, Nucl. Phys. **B454** (1995) 663; T.W.B. Kibble, J. Phys. **A9** (1976) 1387; Phys. Rep. **67** (1980) 183; A. Vilenkin, Phys. Rep. **121** (1985) 263; B. Holdom, Phys. Rev. **D28** (1983) 1419; B. Rai and G. Senjanović, Phys. Rev. **D49** (1994) 2729; S. Weinberg, Phys. Rev. **D9** (1974) 3357; R. N. Mohapatra and G. Sen-

- janovic, Phys. Rev. Lett. **42** (1979) 1651 , Phys. Rev. **D20** (1979) 3390; G. Dvali and G. Senjanović, Phys. Rev. Lett. **74** (1995) 5178; G. Dvali, A. Melfo, and G. Senjanović, Phys. Rev. **D54** (1996) 7857.
- [15] A. Salam and J. Strathdee, Nucl. Phys. **B87** (1975) 85; P. Fayet Nucl. Phys. **B90** (1975) 104; S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150; N. Sakai, Z. Phys. **C11** (1981) 153; H. P. Nilles and S. Raby, Nucl. Phys. **B198** (1982) 102; N. Sakai and T. Yanagida Nucl. Phys. **B197** (1982) 533; S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. **B112** (1982) 133; G. Farrar and S. Weinberg, Phys. Rev. **D27** (1983) 2732; W. Buchmüller and D. Wyler, Phys. Lett. **B121** (1983) 321; R. Barbieri and L. Maiani, Nucl. Phys. **B243** (1984) 429; E. J. Chun, J.E. Kim, and H.P. Nilles, Nucl. Phys. **B370** (1992) 105; J. E. Kim and H. P. Nilles, Mod. Phys. Lett. **A9** (1994) 3575; M. Dine and D. A. MacIntire, Phys. Rev. **D46** (1992) 2594; L. Hall and L. Randall, Nucl. Phys. **B352** (1991) 289; I. Affleck and M. Dine, Phys. Lett. **B154** (1985) 368; R. Barbieri, L. Girardello and A. Masiero, Phys. Lett. **B127** (1983) 429; G. Farrar and A. Masiero, hep-ph/9410401; A.H. Chamseddine and H. Dreiner, Nucl. Phys. **B458** (1996) 65; G. Lazarides and Q. Shafi, Phys. Rev. **D58** (1998) 071702.
- [16] G. Raffelt, Phys. Rep. **198** (1990) 1.
- [17] M. Dine, R. Leigh, and D. McIntire, Phys. Rev. Lett. **69** (1992) 2030; K. Choi, D. Kaplan, and A. Nelson, Nucl. Phys. **B391** (1993) 515.
- [18] D. A. Demir, A. Masiero, and O. Vives, Phys. Rev. **D61**, 075009 (2000); hep-ph/9911337 (to appear in Phys. Lett. B).
- [19] P. Sikivie, Phys. Rev. Lett. **48** (1992) 1156; M.C. Huang and P. Sikivie, Phys. Rev. **D32** (1985) 1560.
- [20] Ya.B. Zel'dovich, I.Yu. Kobzarev and L.B. Okun, Sov. Phys. JETP **40** (1975) 1.
- [21] G. Lazarides and Q. Shafi, Phys. Lett. **115B** (1982) 21; H. Georgi and M. B. Wise, Phys. Lett. **116B** (1982) 123; S. Dimopoulos, P. H. Frampton, H. Georgi, and M. B. Wise, Phys. Lett. **117B** (1982) 185; D. Dominici, R. Holman, and C. W. Kim, Phys. Lett. **130B** (1983) 39.
- [22] D. Chang, P. B. Pal, and G. Senjanovic, Phys. Lett. **153B** (1985) 407; C. Q. Geng and J. N. Ng, Phys. Rev. **D39** (1989) 1449 , Phys. Rev. **D41** (1990) 3848.
- [23] S. Dimopoulos, G. Giudice, and N. Tetradis, Nucl. Phys. **B454** (1995) 59.
- [24] C. Kounnas, F. Zwirner, and I. Pavel, Phys. Lett. **B335** (1994) 403; P. Binetruy and E. Dudas, Phys. Lett. **B338** (1994) 403; C. Kounnas, I. Pavel, G. Ridolfi, and F. Zwirner, Phys. Lett. **B354** (1995) 322; Y. Nambu, preprint EFI 92-37.
- [25] E. Cremmer, S. Ferrara, C. Kounnas, and D. Nanopoulos, Phys. Lett. **B133** (1983) 61; J. Ellis, A. Lahanas, D. Nanopoulos, and K. Tamvakis, Phys. Lett. **B134** (1984) 429; J. Ellis, C. Kounnas, and D. Nanopoulos, Nucl. Phys. **B241** (1984); Nucl. Phys. **B247** (1984) 373; S. Ferrara, C. Kounnas, and F. Zwirner, Nucl. Phys. **B429** (1994) 589; Nucl. Phys. **B433** (1995) 255.