

# Realistic Gluino Axion Model Consistent with Supersymmetry Breaking at the TeV Scale

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## Abstract

The recently proposed model of using the dynamical phase of the gluino to solve the strong CP problem is shown to admit a specific realization in terms of fundamental singlet superfields, such that the breaking of supersymmetry occurs only at the TeV scale, despite the large axion scale of  $10^9$  to  $10^{12}$  GeV. Phenomenological implications are discussed.

The strong CP problem is the problem of having the instanton-induced term [1]

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta} \quad (1)$$

in the effective Lagrangian of quantum chromodynamics (QCD), where  $g_s$  is the strong coupling constant, and

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f_{abc} G_b^\mu G_c^\nu \quad (2)$$

is the gluonic field strength. If  $\theta_{QCD}$  is of order unity, the neutron electric dipole moment is expected [2] to be  $10^{10}$  times its present experimental upper limit ( $0.63 \times 10^{-25} e \text{ cm}$ ) [3]. This conundrum is most elegantly resolved by invoking a dynamical mechanism [4] to relax the above  $\theta_{QCD}$  parameter (including all contributions from colored fermions) to zero. However, this necessarily results [5] in a very light pseudoscalar particle called the axion, which has not yet been observed [6].

To reconcile the nonobservation of an axion in present experiments and the constraints from astrophysics and cosmology [7], two types of “invisible” axions are widely discussed. The DFSZ solution [8] introduces a heavy singlet scalar field as the source of the axion but its mixing with the doublet scalar fields (which couple to the usual quarks) is very much suppressed. The KSVZ solution [9] also has a heavy singlet scalar field but it couples only to new heavy colored fermions.

Consider now the incorporation of supersymmetry into the Standard Model (SM) of particle interactions. The list of colored fermions consists of not only the usual quarks, but also the gluinos. The parameter  $\theta_{QCD}$  of Eq. (1) is then replaced by

$$\bar{\theta} = \theta_{QCD} - \text{Arg Det } M_u M_d - 3 \text{ Arg } M_{\tilde{g}}, \quad (3)$$

where  $M_u$  and  $M_d$  are the respective mass matrices of the charge  $2/3$  and  $-1/3$  quarks, and  $M_{\tilde{g}}$  is the gluino mass. Our recent proposal [10, 11] is to relax  $\bar{\theta}$  to zero with the dynamical phase of the gluino, instead of the quarks as in the DFSZ model or other unknown colored

fermions as in the KSVZ model. The source of this axion is again a heavy singlet scalar field, but since its vacuum expectation value (VEV) is supposed to be in the range  $10^9$  to  $10^{12}$  GeV to satisfy the astrophysical and cosmological bounds [7], supersymmetry is expected to be broken at that scale as well, and not at 1 TeV as desired. This is of course also a problem in the supersymmetric versions of the DFSZ and KSVZ models. In the following we will show how it gets resolved in a specific realization of the gluino axion model [10, 11].

Our *first* key observation is the identification of the anomalous global symmetry  $U(1)_R$  of supersymmetric transformations as the  $U(1)_{PQ}$  symmetry which solves the strong CP problem and generates the axion. Under  $U(1)_R$ , the scalar components of a chiral superfield transform as  $\phi \rightarrow e^{i\theta R}\phi$ , whereas the fermionic components transform as  $\psi \rightarrow e^{i\theta(R-1)}\psi$ . In the minimal supersymmetric standard model (MSSM), the quark and lepton superfields  $\hat{Q}, \hat{u}^c, \hat{d}^c, \hat{L}, \hat{e}^c$  have  $R = +1$ , whereas the Higgs superfields  $\hat{H}_u, \hat{H}_d$  have  $R = 0$ . The superpotential

$$\hat{W} = \mu \hat{H}_u \hat{H}_d + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c + h_e \hat{H}_d \hat{L} \hat{e}^c \quad (4)$$

has  $R = +2$  except for the  $\mu$  term (which has  $R = 0$ ). Hence the resulting Lagrangian breaks  $U(1)_R$  explicitly, leaving only a discrete remnant, i.e. the usual  $R$  parity:  $R = (-1)^{3B+L+2J}$ . The gluino axion model [10, 11] replaces  $\mu$  with a singlet superfield of  $R = +2$  and requires the entire theory to be invariant under  $U(1)_R$ , which is then spontaneously broken. The reason that  $U(1)_R$  is a natural choice for  $U(1)_{PQ}$  is that the gauginos of the MSSM have  $R = +1$ , hence the phase of the gluino mass must be dynamical and contributes to  $\bar{\theta}$  of Eq. (3). In fact, all SM particles have  $R = 0$  (i.e. even  $R$  parity) and all superparticles have  $R = \pm 1$  (i.e. odd  $R$  parity), but the only colored fermions with  $R \neq 0$  are the gluinos. In the minimal SM with only one Higgs doublet, there is no  $U(1)_{PQ}$ , hence both the DFSZ and KSVZ models require additional particles. In the MSSM, there is also no  $U(1)_{PQ}$ , but if the  $\mu$  term and the soft supersymmetry breaking  $A$  terms and gaugino masses are removed, then

the  $U(1)_R$  symmetry is available for us to identify as the  $U(1)_{PQ}$  symmetry for solving the strong CP problem. Of course, we still need to implement this idea with a specific choice of additional particles. However, no matter what we do, we are faced with a fundamental problem (which also exists if we want to consider supersymmetric versions of the DFSZ and KSVZ models): if the superfield containing the axion is spontaneously broken at  $10^9$  to  $10^{12}$  GeV, how is the supersymmetry preserved down to the order of 1 TeV?

Our *second* key observation has to do with the consequence of the spontaneous breaking of a global symmetry without breaking the supersymmetry. Because the supersymmetry is not broken, there has to be a massless superfield, the scalar component of which is *complex*. In addition to the usual phase degree of freedom, there is now also a *scale* degree of freedom, hence such models always contain an indeterminate mass scale [12]. The trick then is to construct a realistic gluino axion model using this ambiguity of scale so that the subsequent soft breaking of supersymmetry occurs at 1 TeV, but the vacuum expectation value of the scalar field containing the axion is  $10^9$  to  $10^{12}$  GeV.

Following Ref.[11], we introduce again three singlet superfields  $\hat{S}_2$ ,  $\hat{S}_1$ , and  $\hat{S}_0$ , with  $R = 2, 1, 0$ , respectively and impose the  $Z_3$  discrete symmetry under which  $\hat{S}_1$  and  $\hat{S}_0$  transform as  $\omega$  and  $\hat{S}_2$  as  $\omega^2$ , with  $\omega^3 = 1$ . The most general superpotential with  $R = 2$  containing these superfields is then given by

$$\hat{W} = m_2 \hat{S}_2 \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0. \quad (5)$$

Let  $\hat{S}_{2,1,0}$  be replaced by  $v_{2,1,0} + \hat{S}_{2,1,0}$ , then

$$\begin{aligned} \hat{W} &= m_2 v_0 \hat{S}_2 + 2f_1 v_1 v_0 \hat{S}_1 + (m_2 v_2 + f_1 v_1^2) \hat{S}_0 \\ &+ f_1 v_0 \hat{S}_1 \hat{S}_1 + (m_2 \hat{S}_2 + 2f_1 v_1 \hat{S}_1) \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0. \end{aligned} \quad (6)$$

Hence the minimum of the corresponding scalar potential is given by

$$V_{min} = |m_2 v_0|^2 + 4|f_1 v_1 v_0|^2 + |m_2 v_2 + f_1 v_1^2|^2. \quad (7)$$

To preserve supersymmetry, we need  $V_{min} = 0$ . Hence

$$v_0 = 0, \quad v_2 = -\frac{f_1 v_1^2}{m_2}. \quad (8)$$

If  $v_{1,2} \neq 0$ , then  $U(1)_R$  is spontaneously broken, but the scale of symmetry breaking is indeterminate [12] because only the ratio  $v_1^2/v_2$  is constrained. Moreover, since  $m_2$  is presumably very large, say of the order of some unification scale,

$$v_2 \ll v_1 \quad (9)$$

is predicted, unless of course both  $v_1$  and  $v_2$  are of order  $m_2$ . (Exactly what value each actually takes will depend on the subsequent soft breaking of the supersymmetry as we will show later.) With this solution,

$$\hat{W} = \frac{m_2}{v_1}(v_1 \hat{S}_2 - 2v_2 \hat{S}_1) \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0, \quad (10)$$

which shows clearly that the linear combination

$$\frac{v_1 \hat{S}_1 + 2v_2 \hat{S}_2}{\sqrt{|v_1|^2 + 4|v_2|^2}} \quad (11)$$

is a massless superfield. Hence the axion is mostly contained in  $S_1$ , but since only  $S_2$  couples to the MSSM particles, the effective axion coupling to gluinos is  $(v_2/v_1)v_2^{-1} = v_1^{-1}$  as desired.

Consider now the breaking of the supersymmetry by soft terms at the TeV scale which preserve the  $U(1)_R$  symmetry but are allowed to break the  $Z_3$  discrete symmetry [13]. We start with the original superpotential of Eq. (5), write down its corresponding scalar potential, and add all such soft terms regardless of whether or not they are holomorphic, i.e.

$$\begin{aligned} V &= |m_2 S_0|^2 + 4|f_1 S_0 S_1|^2 + |m_2 S_2 + f_1 S_1^2|^2 \\ &+ \mu_0^2 |S_0|^2 + \mu_1^2 |S_1|^2 + \mu_2^2 |S_2|^2 \\ &+ [\mu_{12} S_1^2 S_2^* + \mu_{00} S_0 |S_0|^2 + \mu_{01} S_0 |S_1|^2 + \mu_{02} S_0 |S_2|^2 + h.c.] \end{aligned} \quad (12)$$

The minimum of  $V$  is now determined by

$$v_0 \simeq -\frac{\mu_{01}v_1^2}{m_2^2}, \quad (13)$$

$$v_1^2 \simeq \frac{\mu_1^2 m_2}{4f_1\mu_{12}} \left[ 1 - \frac{\mu_{12}}{2f_1 m_2} + \frac{f_1\mu_2^2}{2m_2\mu_{12}} + \frac{\mu_2^2}{m_2^2} \right], \quad (14)$$

$$v_2 \simeq -\frac{\mu_1^2}{4\mu_{12}} \left[ 1 - \frac{\mu_{12}}{f_1 m_2} + \frac{f_1\mu_2^2}{2m_2\mu_{12}} \right]. \quad (15)$$

Hence  $v_0 \ll v_2 \ll v_1$  and the supersymmetric solution of Eq. (8) remains valid to a very good approximation. (In Ref.[11],  $\mu_{12}$  was written as  $\lambda m_2$  with the implicit assumption that it is of order  $m_2$ , hence  $\mu_1$  in that case is of order  $v_1$ .) We now realize the important fact that all soft supersymmetry breaking parameters ( $\mu_1, \mu_{12}, etc.$ ) can be of order 1 TeV so that  $v_2$  is of order 1 TeV as shown by Eq. (15), and yet  $v_1$  is larger than  $v_2$  by a factor of order  $\sqrt{m_2/v_2}$ .

Consider now the physical masses of  $S_{2,1,0}$  and their fermionic partners. The linear combination given by Eq. (11) still contains the axion, but because supersymmetry (as well as  $Z_3$ ) is broken at the TeV scale, its fermionic component (axino) is allowed to have a Majorana mass of that magnitude. The phase of its bosonic component is the axion, but the magnitude (maxion) is a scalar field of mass  $\sqrt{-2\mu_1^2}$  (instead of zero). The orthogonal combination to that given by Eq. (11) combines with  $\hat{S}_0$  to form a heavy superfield containing two complex scalars and a Dirac fermion of mass  $m_2$  as expected. Hence the low-energy particle content of our model consists of (i) a Majorana fermion at the TeV scale, (ii) a real scalar field also at the TeV scale, and (iii) an axion which couples only to superparticles. Since all the above particles come from mostly  $\hat{S}_1$  but only  $\hat{S}_2$  interacts directly with the MSSM particles, their effects are generally suppressed by the factor  $v_2/v_1$ .

Our *third* key observation has to do with how gauginos acquire mass in the presence of  $U(1)_R$ . In Refs.[10, 11], the explicit arbitrary supersymmetry-breaking term  $S_2^* \tilde{g} \tilde{g}$  is assumed. Since this is not a soft term, it is not clear how it can be justified rigorously. Here

we show that it is actually generated through loop corrections. The so-called  $A$  terms are also generated through their effective couplings to  $S_2$ . Both gaugino masses and  $A$  terms are forbidden by  $U(1)_R$  invariance, but are allowed as  $v_1$  and  $v_2$  become nonzero.

The MSSM superpotential of Eq. (4) is now replaced by

$$\begin{aligned}\hat{W} &= h_2 \hat{S}_2 \hat{H}_u \hat{H}_d + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c + h_e \hat{H}_d \hat{L} \hat{e}^c \\ &+ m_2 \hat{S}_2 \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0,\end{aligned}\tag{16}$$

where we have assumed that  $\hat{H}_u, \hat{H}_d$  transform as  $\omega^2$ ,  $\hat{Q}, \hat{L}$  as  $\omega$  and  $\hat{u}^c, \hat{d}^c, \hat{e}^c$  as 1 under  $Z_3$ . The breaking of supersymmetry is achieved by the soft terms of Eq. (12) together with

$$\begin{aligned}V_{soft} &= \tilde{Q}^\dagger M_Q^2 \tilde{Q} + \tilde{u}^{c\dagger} M_{u^c}^2 \tilde{u}^c + \tilde{d}^{c\dagger} M_{d^c}^2 \tilde{d}^c + \tilde{L}^\dagger M_L^2 \tilde{L} + \tilde{e}^{c\dagger} M_{e^c}^2 \tilde{e}^c \\ &+ M_{H_u}^2 |H_u|^2 + M_{H_d}^2 |H_d|^2 + (M_{ud}^2 H_u H_d + h.c.)\end{aligned}\tag{17}$$

This differs from that of the MSSM only in that the  $A$  terms and the gaugino masses are absent because they are not invariant under  $U(1)_R$ . The usual  $\mu B$  term of the MSSM is denoted as  $M_{ud}^2$  here because the parameter  $\mu$  is now absent.

From Eq. (16), we find the following terms,

$$|h_2 S_2 H_d + h_u \tilde{Q} \tilde{u}^c|^2 + |m_2 S_0 + h_2 H_u H_d|^2,\tag{18}$$

in the Lagrangian of our model. Together with the  $H_u H_d$  term in Eq. (17), we obtain an effective interaction (see Fig. 1, left window) given by

$$\left( M_{ud}^2 + \frac{h_2}{f_1} v_2 \mu_{01} \right) \frac{h_2^* h_u}{M_{H_d}^2} S_2^* H_u \tilde{Q} \tilde{u}^c,\tag{19}$$

where Eqs. (13) to (15) have been used. This means that an effective  $A$  term is generated as  $S_2$  is replaced by its VEV, i.e.

$$A_u = \frac{h_2^* h_u v_2}{M_{H_d}^2} \left( M_{ud}^2 + \frac{h_2}{f_1} v_2 \mu_{01} \right),\tag{20}$$

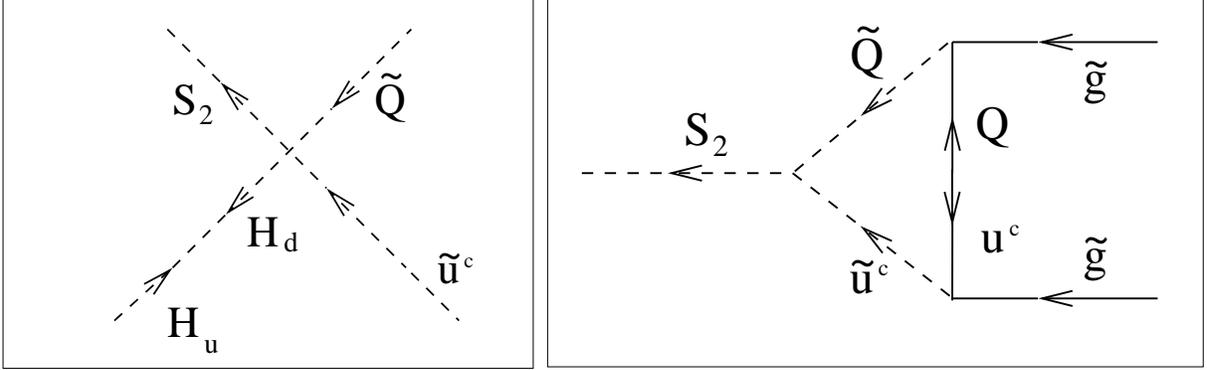


Figure 1: The diagrams that generate the triscalar coupling  $A_u$  (left), and the gluino mass (right).

which has the desirable feature of being proportional to  $h_u$  and thus the automatic suppression of flavor-changing neutral currents from the supersymmetric scalar sector.

Using Eq. (18), we also obtain the effective interaction (see Fig. 1, right window) given by

$$\frac{g_s^2}{16\pi^2} h_2^* h_u \frac{v_d m_u}{M_{eff}^2} S_2^* \tilde{g} \tilde{g}, \quad (21)$$

which generates a gluino mass proportional to  $v_2$ , together with a dynamical phase from  $S_2$ . This solves the strong CP problem as proposed in Ref.[10]. However, the generated mass itself is very small. Even with  $m_t = 174$  GeV,  $M_{\tilde{g}}$  is at most a few GeV. On the other hand, such a light gluino is not completely ruled out experimentally and may yet be discovered [14]. To obtain a heavy gluino (with mass greater than 250 GeV), some new physics at the TeV scale will be required. For example, consider the addition of a neutral singlet  $\hat{\chi}$  with  $R = 0$  and colored triplets  $\hat{\psi}$  and  $\hat{\psi}^c$  with  $R = 1$ . All are assumed to transform as  $\omega^2$  under  $Z_3$ . Then the extra terms in the superpotential, i.e.

$$f_2 \hat{S}_2 \hat{\chi} \hat{\chi} + f_0 \hat{\chi} \hat{\psi} \hat{\psi}^c, \quad (22)$$

will generate a gluino mass given by

$$M_{\tilde{g}} = \frac{g_s^2}{8\pi^2} f_2 v_2, \quad (23)$$

where the masses of  $\tilde{\psi}$ ,  $\tilde{\psi}^c$  and the Dirac fermion formed out of  $\psi$  and  $\psi^c$  are all taken to be  $f_0\langle\chi\rangle$ . In Eq. (15), let  $|\mu_1| = 4$  TeV,  $\mu_{12} = 0.4$  TeV, then  $v_2 = 10$  TeV. Now let  $f_2 = 1.3$ , then  $M_{\tilde{g}} = 250$  GeV.

The usual incorporation of the axion into a supersymmetric model assumes the original Peccei-Quinn symmetry [4] for the corresponding superfields, i.e.  $+1/2$  for  $\hat{Q}$ ,  $\hat{u}^c$ ,  $\hat{d}^c$ ,  $\hat{L}$ ,  $\hat{e}^c$ , and  $-1$  for  $\hat{H}_u$ ,  $\hat{H}_d$ . This assignment forbids the  $\mu$  term in the MSSM superpotential, as well as the  $B$  term in  $V_{soft}$ . Assuming supergravity, the  $U(1)_{PQ}$  symmetry is then broken together with local supersymmetry in the Kähler potential at the axion scale  $f_a$ . The effective scale of global supersymmetry breaking becomes of order  $f_a^2/M_{Planck}$ . The singlet superfield carrying the axion is here some kind of ‘‘messenger’’ field which communicates between the MSSM and the hidden sector. In our case, the axion comes from a superfield which lives entirely in our world. The origin of soft supersymmetry breaking at the TeV scale is not specified, only that it has to preserve  $U(1)_R$ . Given such a structure and with the help of a  $Z_3$  discrete symmetry which is softly broken also at the TeV scale, we find that the  $U(1)_R$  symmetry is actually broken spontaneously at a scale much larger than  $M_{SUSY}$ . In fact, this mechanism also works if we use the original  $U(1)_{PQ}$  instead of  $U(1)_R$ . In that case,  $\hat{S}_{2,1,0}$  should have  $PQ$  charges of  $+2$ ,  $-1$ , and  $-2$  respectively. Hence the superpotential of Eq. (8) is obtained without using the  $Z_3$  discrete symmetry [15].

In conclusion, we have succeeded in formulating a realistic supersymmetric model with a spontaneously broken  $U(1)_R$  symmetry as the natural solution of the strong CP problem. The soft breaking of the supersymmetry at the TeV scale induces an axion scale of order  $\sqrt{m_2 M_{SUSY}}$ , where  $m_2$  is some unification scale, such as the string scale or the Planck scale. We have thus a first example of the unusual situation where the mass of the physical field ( $S_1$ ) is much smaller than its VEV. This is in contrast to the less uncommon occurrence [16] where the mass of the physical field ( $S_2, S_0$ ) is much greater than its VEV.

The resulting model resembles closely the MSSM, with the following distinctions. (1) The  $\mu$  parameter is replaced by  $h_2\hat{S}_2$  in the superpotential, thus solving the so-called  $\mu$  problem. (2) Although the breaking of  $U(1)_R$  by  $v_1$  and  $v_2$  also breaks  $R$  parity, the latter is effectively conserved as far as the MSSM particles are concerned because its violation is suppressed by  $v_2/v_1$ . (3) Supersymmetric scalar masses and the equivalent  $B$  term are as in the MSSM. However,  $U(1)_R$  invariance forbids  $A$  terms and gaugino masses. (4) The spontaneous breaking of  $U(1)_R$  leads to  $A$  terms proportional to  $v_2$  and to the corresponding Yukawa coupling matrix, thereby suppressing flavor-changing neutral currents automatically. (5) Gaugino masses are generated radiatively but are probably too small to be realistic. They can be made larger by the addition of new particles at the TeV scale. (6) In contrast to the DFSZ and KSVZ models, the new particles associated with the axion, i.e. those of Eq. (11), are now at the TeV scale. However, their couplings with the MSSM particles are all suppressed by  $v_2/v_1$ , so they are effectively unobservable at future colliders [17].

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