

Correlating μ Parameter and Right-Handed Neutrino Masses in $N = 1$ Supergravity

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Abstract

The minimal supersymmetric standard model, when extended to embed the seesaw mechanism, obtains two dimensionful parameters in its superpotential: the μ parameter and the right-handed neutrino mass M_N . These mass parameters, belonging to the supersymmetric sector of the theory, pose serious naturalness problems as their scales are left completely undetermined. In fact, for correct phenomenology, μ must be stabilized at the electroweak scale while M_N lies at an intermediate scale. In this work we construct an explicit model of the hidden sector of $N = 1$ supergravity for inducing both μ and M_N at their right scales. The model we build utilizes lepton number conservation and continuous R invariance as two fundamental global symmetries to forbid bare μ and M_N appearing in the superpotential, and induces them at phenomenologically desired scales via spontaneous breakdown of the global symmetries and the supergravity. We discuss briefly various phenomenological implications of the model.

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Contents

1	Introduction	1
2	The Model	3
3	Phenomenological Implications	9
4	Conclusion	15
5	Acknowledgements	16

1 Introduction

Supergravity, once spontaneously broken at a scale M_{susy} , gives rise to a softly-broken globally supersymmetric theory at a scale M_{susy}^2/M_{Pl} which corresponds to the mass scale of the gravitino, $m_{3/2}$ [2] (notice that M_{susy} differs from soft masses often denoted by m_{SUSY}). If the gravitino mass is at the weak scale, $m_{3/2} \sim (1 - 10)M_{EW}$, or equivalently, if supergravity is spontaneously broken at an intermediate scale $M_{\text{susy}}^2 \sim m_{3/2}M_{Pl}$, the gauge hierarchy problem is solved: The fact that supersymmetry is broken only softly guarantees that the electroweak scale is radiatively stable, that is, the ratio $M_{EW}/M_{Pl} \sim m_{3/2}/M_{Pl}$ is immunized against quantum fluctuations.

The most economic description of the observable sector is realized by the Minimal Supersymmetric Standard Model (MSSM) which essentially corresponds to a direct supersymmetrization of the SM spectrum. One of the crucial aspects of the entire mechanism is the breakdown of local supersymmetry at an intermediate scale $M_{\text{susy}}^2 \sim m_{3/2}M_{Pl}$ which itself poses a new naturalness problem. The question of how such an intermediate scale has been formed can be answered only through a concrete modeling of the hidden sector. Just to give an idea, one can consider, for instance, dynamical supersymmetry breaking scenarios in which all energy scales in the infrared are generated from M_{Pl} via dimensional transmutation [3]. Right here one recalls that intermediate scales like M_{susy} are also necessitated by other phenomena not related to supersymmetry breaking. As an example, one can allude

to the Peccei-Quinn mechanism [6] which is devised to solve the strong CP problem. This mechanism is based on the presence of an intermediate scale $M_{PQ} \sim 10^{14}$ GeV.

Another example, on which we are going to concentrate in this paper, is the famous seesaw mechanism [5] which explains the tiny but nonzero neutrino masses. The seesaw mechanism is based on the existence of ultra heavy right-handed neutrinos, N_i , which are singlets of the $SU(3) \times SU(2) \times U(1)$ symmetry of the standard model. Indeed, in spite of several alternative models [4], the seesaw mechanism is arguably the most popular way of explaining the tiny masses of neutrinos, partly because it provides an explanation for the baryon asymmetry of the universe through a mechanism called leptogenesis [7]. Successful leptogenesis requires masses of heavy right-handed neutrinos to be above $\sim 10^9$ GeV [8].

Obviously, low-energy phenomena do not necessitate any correlation among the mass scales M_{SUSY} , M_{PQ} and M_N . They show up as independent scales, needed to explain distinct phenomena. However, it would establish a strong case, besides superstrings, for the existence of a supersymmetric organizing principle operating at ultra high energies if they can be correlated within a specific model. Concerning this point, one here recalls [9] in which M_{PQ} and M_{SUSY} have been correlated by using a hidden sector composite axion. The subject matter of the present work will be essentially to relate M_{SUSY} and M_N , leaving aside M_{PQ} , within $N = 1$ supergravity.

The superpotential of MSSM-RN is given by

$$\begin{aligned} \widehat{W}_{\text{MSSM-RN}} &= \mu \widehat{H}_u \cdot \widehat{H}_d + M_N \widehat{N}^c \widehat{N}^c \\ &+ Y_u \widehat{Q} \cdot \widehat{H}_u \widehat{U}^c + Y_d \widehat{H}_d \cdot \widehat{Q} \widehat{D}^c + Y_e \widehat{H}_d \cdot \widehat{L} \widehat{E}^c + Y_\nu \widehat{L} \cdot \widehat{H}_u \widehat{N}^c \end{aligned} \quad (1)$$

where \widehat{N}^c stands for the anti right-handed neutrino supermultiplet [5]. This model classically preserves the baryon number while R parity and SM gauge group are exact symmetries of the model even at the quantum level.

The superpotential (1) involves two dimensionful parameters μ and M_N . These mass parameters are both nested in the superpotential, and thus, they bear no relation whatsoever to the supersymmetry breaking sector of the theory. Therefore, they pose a serious naturalness problem in that MSSM offers no mechanism, dynamical or otherwise, to enable one to know characteristic scales of μ and M_N . In fact, present neutrino data already require $M_N \sim \langle H_u \rangle^2 Y_\nu^2 / m_\nu \gtrsim 10^{14} (Y_\nu)^2$ GeV. On the other hand, LEP lower bound on chargino

mass [10] leads in a rather model independent way to a lower bound on the μ parameter: $\mu > 110$ GeV [11]. On the other hand, to have a successful electroweak symmetry breaking μ has to be stabilized at the weak scale [12]. Consequently, the MSSM must be extended to provide a dynamical understanding of how and why μ and M_N are stabilized to their phenomenologically favored scales. This is the naturalness problem we discuss in this work. Actually, part of the problem *i.e.* stabilization of the μ parameter at the TeV scale has already been discussed in various contexts and various solutions have been devised [12]. What is left over is to understand the mechanism which stabilizes M_N at an intermediate scale of $\sim 10^{14}$ GeV. In what follows we will attack on both naturalness problems by constructing a hidden sector model which leads to a dynamical determination of μ and M_N upon breakdown of certain global symmetries [13] and supergravity [14]. It will turn out to be a hybrid model in that we will utilize both Kähler potential and superpotential of the hidden sector fields such that induction of the μ parameter proceeds in a way similar to the Giudice-Masiero mechanism [14].

In Sec. 2, we will attempt to formulate a model of hidden sector dynamics which induces μ and M_N *simultaneously*, as a result of spontaneous local supersymmetry breaking. As will be seen, this can be accomplished by including logarithmic terms in the Kähler potential (which might be motivated by string theory [15]). In Sec. 3, we will provide a brief discussion of certain phenomenological implications of this model. In Sec. 4, we summarize our conclusions.

2 The Model

Any attempt at answering the question put forward in the introduction should first provide a way of forbidding bare μ and M_N appearing in the superpotential. The fact that $\widehat{W}_{MSSM-RN}$ possesses additional continuous symmetries in the absence of μ and M_N [13] implies that imposing global symmetries can protect $\widehat{W}_{MSSM-RN}$ against the bare μ and M_N parameters. In particular, imposing a global continuous R invariance and restoring lepton number conservation forbid the bare μ and M_N parameters, respectively. This can be achieved by introducing new fields appropriately charged under these symmetries. Realization of this setup requires at least two hidden sector chiral superfields $\widehat{z}_1, \widehat{z}_2$ with charges $R(\widehat{z}_1) = 0$,

$R(\widehat{z}_2) = 2$, $L(\widehat{z}_1) = 2$ and $L(\widehat{z}_2) = -2$. The R charge of each lepton and quark superfield equals 1 and $R(\widehat{H}_u) = R(\widehat{H}_d) = 0$ so that the superpotential (1) acquires two units of R charge. The complete superpotential can be decomposed as

$$\widehat{W} = \widehat{W}_{obs} + \widehat{W}_{hid} + \widehat{W}_{obs-hid} \quad (2)$$

where

$$\begin{aligned} \widehat{W}_{obs} &= Y_u \widehat{Q} \cdot \widehat{H}_u \widehat{U}^c + Y_d \widehat{H}_d \cdot \widehat{Q} \widehat{D}^c + Y_e \widehat{H}_d \cdot \widehat{L} \widehat{E}^c + Y_\nu \widehat{L} \cdot \widehat{H}_u \widehat{N}^c \\ \widehat{W}_{hid} &= M_{hid} \widehat{z}_2 \widehat{z}_1 \\ \widehat{W}_{obs-hid} &= \lambda \widehat{z}_1 \widehat{N}^c \widehat{N}^c / 2 \end{aligned} \quad (3)$$

where, like M_N , λ is a matrix in the space of right-handed neutrino flavors. This superpotential respects a global $U(1)_{R-sym} \otimes U(1)_{Lepton}$ invariance in addition to the baryon number conservation and the MSSM gauge symmetries.

The supergravity Lagrangian is based on the Kähler potential [2]

$$\mathcal{G}(\widehat{\phi}, \widehat{\phi}^\dagger) = K(\widehat{\phi}, \widehat{\phi}^\dagger) + M_{Pl}^2 \ln \left| \frac{\widehat{W}(\widehat{\phi})}{M_{Pl}^3} \right|^2 \quad (4)$$

where $\widehat{\phi} = (\widehat{z}_i; \widehat{H}_u, \widehat{H}_d, \dots, \widehat{N}^c)$ collectively denotes the chiral superfields in the hidden and observable sectors of the theory. The kinetic terms of the superfields are collected in $K(\widehat{\phi}, \widehat{\phi}^\dagger)$. The part of the scalar potential induced by F -terms is given by

$$V_F(\phi) = e^{K/M_{Pl}^2} \left\{ g^{ab^*} D_a W D_{b^*} W^* - 3 \frac{|W|^2}{M_{Pl}^2} \right\} \quad (5)$$

where

$$D_a W = \frac{\partial W}{\partial \phi^a} + \frac{W}{M_{Pl}^2} \frac{\partial K}{\partial \phi^a} \quad (6)$$

and g_{ab^*} is the Kähler metric:

$$g_{ab^*} = \frac{\partial^2 K}{\partial \phi^a \partial \phi^{*b}}. \quad (7)$$

In addition to V_F , there are contributions from D -terms as well. However, D -terms do not contribute to supersymmetry breaking since, in our model, none of the hidden sector superfields exhibits a gauge invariance.

Similarly to \widehat{W} , we decompose kinetic terms of the superfields as

$$\begin{aligned}
K_{obs} &= \widehat{H}_u^\dagger \widehat{H}_u + \widehat{H}_d^\dagger \widehat{H}_d + \dots + \widehat{N}^{c\dagger} \widehat{N}^c \\
K_{hid} &= \widehat{z}_1^\dagger \widehat{z}_1 + \widehat{z}_2^\dagger \widehat{z}_2 + C_1 M_{Pl}^2 \log \frac{\widehat{z}_1^\dagger \widehat{z}_1}{M_{Pl}^2} + C_2 M_{Pl}^2 \log \frac{\widehat{z}_2^\dagger \widehat{z}_2}{M_{Pl}^2} \\
K_{obs-hid} &= \frac{1}{M_{Pl}^2} \left(\lambda_1 \widehat{z}_1^\dagger \widehat{z}_1 + \lambda_2 \widehat{z}_2^\dagger \widehat{z}_2 \right) \widehat{H}_u \cdot \widehat{H}_d + \text{h.c.}
\end{aligned} \tag{8}$$

where $K_{obs-hid}$ is similar to the operator used in the Giudice-Masiero mechanism [14] which solves the naturalness problem associated with the μ parameter[‡]. The logarithmic terms in K_{hid} , which might be inspired from strings [15], do not change the Kähler metric and are included to achieve a sensible vacuum configuration in the hidden sector[§]. The dimensionless couplings C_1 and C_2 are determined from the minimization of (5) and demanding zero (or very small) cosmological constant.

The scalar potential of the hidden sector fields (5) takes the form

$$\begin{aligned}
V_F(z_1, z_2) &= M_{hid}^2 \exp \left[\frac{|z_1|^2 + |z_2|^2}{M_{Pl}^2} \right] \left[\frac{|z_1|^2}{M_{Pl}^2} \right]^{C_1} \left[\frac{|z_2|^2}{M_{Pl}^2} \right]^{C_2} \\
&\times \left[|z_1|^2 \left(1 + C_2 + \frac{|z_2|^2}{M_{Pl}^2} \right)^2 + |z_2|^2 \left(1 + C_1 + \frac{|z_1|^2}{M_{Pl}^2} \right)^2 - 3 \frac{|z_1|^2 |z_2|^2}{M_{Pl}^2} \right]
\end{aligned} \tag{9}$$

[‡]In principle, the Kähler potential can include higher order terms such as

$$\Delta K_{obs-hid} = \sum_{m,n>1} \frac{\beta_{mn}}{(m!n!)^2} \left(\frac{\widehat{z}_1^\dagger \widehat{z}_1}{M_{Pl}^2} \right)^m \left(\frac{\widehat{z}_2^\dagger \widehat{z}_2}{M_{Pl}^2} \right)^n \widehat{H}_u \cdot \widehat{H}_d$$

where β_{mn} are dimensionless constants. In our analysis, we restrict ourselves to the minimal case and neglect such higher order effects noticing that they do not alter the scale of observable sector parameters *e.g.* the μ parameter and Higgs bilinear term to be derived in this section.

[§]In general, quantum gravitational interactions do not respect global symmetries [16], and thus, Kähler potential above can receive corrections of the form

$$\Delta K_{hid} = \sum_{m,n,p,q>2} \frac{\alpha_{mnpq}}{M_{Pl}^{m+n+p+q-2}} \frac{(\widehat{z}_1^\dagger)^m \widehat{z}_1^n (\widehat{z}_2^\dagger)^p \widehat{z}_2^q}{m!n!p!q!}$$

where α_{mnpq} are dimensionless constants. Recalling the fact that global symmetry breaking effects get strongly suppressed if gravity is modified near the Planckian scale [17], throughout this work we neglect such terms.

where $z_{1,2}$ stand for the scalar components of $\widehat{z}_{1,2}$, respectively. (In what follows, we will denote their fermionic partners by $\psi_{z_{1,2}}$.) Clearly, V_F diverges as $|z_1|, |z_2| \rightarrow \infty$. Moreover, when $C_1, C_2 < 0$ and $C_1 + C_2 < -1$ potential is not minimized for vanishing z_1 and z_2 . For determining the vacuum configuration, we should solve

$$\frac{\partial V_F}{\partial |z_1|^2} = 0, \quad \frac{\partial V_F}{\partial |z_2|^2} = 0, \quad V_F(z_1, z_2) = 0 \quad (10)$$

where the first two determine the extremum of the potential whereas the third is needed for nullifying the cosmological constant. These conditions lead to the constraints

$$1 + C_2 = \frac{|\langle z_2 \rangle|^2}{M_{Pl}^2}, \quad 1 + C_1 = \frac{|\langle z_1 \rangle|^2}{M_{Pl}^2}, \quad |\langle z_1 \rangle|^2 + |\langle z_2 \rangle|^2 = \frac{3}{4} M_{Pl}^2 \quad (11)$$

where F components of $\widehat{z}_{1,2}$ also develop VEVs

$$\begin{aligned} \langle F_{z_1} \rangle &= 2M_{hid}^* \langle z_2^* \rangle \frac{|\langle z_1 \rangle|^2}{M_{Pl}^2} \exp \left[\frac{K}{2M_{Pl}^2} \right] \sim M_{Pl} M_{hid} \\ \langle F_{z_2} \rangle &= 2M_{hid}^* \langle z_1^* \rangle \frac{|\langle z_2 \rangle|^2}{M_{Pl}^2} \exp \left[\frac{K}{2M_{Pl}^2} \right] \sim M_{Pl} M_{hid}. \end{aligned} \quad (12)$$

The vacuum configuration is symmetric under simultaneous ($C_1 \leftrightarrow C_2$) and ($|\langle z_1^2 \rangle| \leftrightarrow |\langle z_2^2 \rangle|$) exchanges. Clearly, C_1 and C_2 have to add up to $-5/4$. The vanishing of the cosmological constant puts stringent constraints on the allowed ranges of C_1 and C_2 . Indeed, in order to have a nontrivial vacuum with $\langle z_{1,2} \rangle, \langle F_{z_{1,2}} \rangle \neq 0$ and with vanishing energy, one needs $-1 < C_2 < -1/4$. (Notice that if we set V_F nonzero but equal to an exceedingly small value corresponding to the observed cosmological constant, C_1, C_2 and $\langle z_{1,2} \rangle$ get modified only slightly, leaving the overall argument similar to the case $V_F = 0$.) As an explicit example, let us consider the case $(C_1, C_2) = (-5/8, -5/8)$ which gives rise to $|\langle z_1^2 \rangle| = |\langle z_2^2 \rangle| = (3/8)M_{Pl}^2$ at which $V(z_1, z_2) = 0$ as desired, and the matrix $M_{i,j} = \partial^2 V_F(z_1, z_2) / \partial |z_i|^2 \partial |z_j|^2$ acquires positive eigenvalues (5.41, 5.41), guaranteeing thus the minimization of the potential. (Obviously, there is nothing special about these numerical values we assign to C_1 and C_2 ; they are picked up just for a fast analysis of the potential landscape.) Another example is $(C_1, C_2) = (-75/76, -5/19)$ which yields $|\langle z_1^2 \rangle| = 0.013M_{Pl}^2$ and $|\langle z_2^2 \rangle| = 0.737M_{Pl}^2$ at which $V(z_1, z_2) = 0$ as desired, and M_{ij} develops positive eigenvalues (242.71, 4.34). These case studies illustrate the behavior of the potential landscape as a function of C_1 and C_2 .

We can rewrite the F -terms in (12) as

$$\begin{aligned}\langle F_{z_1} \rangle &= 2e^{3/8} (1 + C_1)^{1+\frac{1}{2}C_1} (1 + C_2)^{\frac{1}{2}C_2} \times \langle z_2^* \rangle \times M_{hid}^* \\ \langle F_{z_2} \rangle &= 2e^{3/8} (1 + C_1)^{\frac{1}{2}C_1} (1 + C_2)^{1+\frac{1}{2}C_2} \times \langle z_1^* \rangle \times M_{hid}^*\end{aligned}\quad (13)$$

which are $\mathcal{O}(M_{hid}M_{Pl})$. Indeed, for $(C_1, C_2) = (-5/8, -5/8)$ it turns out that $|\langle F_{z_1} \rangle| = |\langle F_{z_2} \rangle| \approx 1.25M_{hid}M_{Pl}$, and for $(C_1, C_2) = (-75/76, -5/19)$ it is that $|\langle F_{z_1} \rangle| \approx 0.30M_{hid}M_{Pl}$ and $|\langle F_{z_2} \rangle| \approx 2.16M_{hid}M_{Pl}$. In the latter, $|\langle F_{z_1} \rangle|$ is smaller than $|\langle F_{z_2} \rangle|$ by an order of magnitude due to relative smallness of $|\langle z_1^2 \rangle|$. The gravitino mass obeys the relation

$$m_{3/2}^2 = M_{Pl}^2 e^{\mathcal{G}/M_{Pl}^2} = e^{3/4} (1 + C_1)^{1+C_1} (1 + C_2)^{1+C_2} M_{hid}^2 \quad (14)$$

which yields $m_{3/2} \sim M_{hid}$. Indeed, it gives $m_{3/2} = M_{hid}$ for $(C_1, C_2) = (-5/8, -5/8)$, and $m_{3/2} \approx 1.26M_{hid}$ for $(C_1, C_2) = (-75/76, -5/19)$. Consequently, the mass parameter M_{hid} in \widehat{W}_{hid} corresponds to the gravitino mass. In other words, $M_{hid} \simeq m_{3/2}$ in the superpotential and the fundamental scale of gravity M_{Pl} combine to break supersymmetry at the intermediate scale $M_{\text{SUSY}}^2 \sim M_{hid}M_{Pl}$ as suggested by the sizes of the associated F -terms (13).

The spontaneous breakdown of local supersymmetry induces soft breakdown of global supersymmetry in the observable sector (presumably with additional hidden sector fields different from \widehat{z}_i which facilitate induction of μ and M_N in the present model) such that each of the scalar fields acquires a mass-squared $\sim m_{3/2}^2$ and each Yukawa interaction in \widehat{W}_{obs} gives rise to a triscalar coupling $\sim m_{3/2}$ [2].

Concerning the parameters pertaining to the right-handed neutrino sector, from Eq. (3), one finds that the right-handed neutrinos acquire a mass term in the superpotential

$$\frac{M_N}{2} \widehat{N}^c \widehat{N}^c \quad \text{where} \quad M_N = \lambda' \langle z_1 \rangle \quad (15)$$

and

$$\lambda' \equiv e^{\frac{K}{2M_{Pl}^2}} \lambda = e^{3/8} (1 + C_1)^{C_1/2} (1 + C_2)^{C_2/2} \lambda \quad (16)$$

in accord with the fact that, in supergravity framework, observable Yukawa couplings are related to the bare ones in the superpotential via Kähler dressing. Moreover, scalar right-handed neutrinos acquire the bilinear coupling

$$\frac{B_N}{2} \widetilde{N}^c \widetilde{N}^c \quad \text{where} \quad B_N = \lambda' \langle F_{z_1} \rangle. \quad (17)$$

In terms of λ' , $M_N \approx 0.61\lambda' M_{Pl}$ and $B_N \approx 1.25\lambda' M_{hid} M_{Pl}$ for $(C_1, C_2) = (-5/8, -5/8)$, and $M_N \approx 0.11\lambda' M_{Pl}$ and $B_N \approx 0.30\lambda' M_{hid} M_{Pl}$ for $(C_1, C_2) = (-75/76, -5/19)$. The model, in general, predicts

$$B_N M_N^{-1} = 2m_{3/2} \quad (18)$$

from which it follows that B_N falls in a range that is too large for “soft leptogenesis” to be effective [18]. On the other hand, B_N is too small for inducing significant radiative effects [19]. These numerical estimates are intended for consistency check of the model.

The seesaw-induced neutrino masses are given by [5]

$$m_\nu = Y'_\nu{}^T M_N^{-1} Y'_\nu \langle H_u \rangle^2. \quad (19)$$

where again $Y'_\nu = e^{\frac{K}{2M_{Pl}^2}} Y_\nu$. One finds that in order to have $m_\nu \simeq 0.05$ eV, for $|Y'_\nu| \lesssim 1$, the right-handed neutrino masses must satisfy $M_N \lesssim 10^{15}$ GeV. This implies that $(0.1 - 1)\lambda' \lesssim 10^{-3}$ or $\lambda' \lesssim 10^{-2}(0.1 - 1)$. For the neutrino Yukawa matrix $Y'_\nu \sim \mathcal{O}(1)$, M_N reaches 10^{15} GeV level and hence λ' takes its maximal value.

With the minimal form of the Kähler potential given in (8), the effective μ parameter of the MSSM is generated to be

$$\begin{aligned} \mu &= \frac{1}{M_{Pl}^2} \left(\lambda_1 \langle z_1 \rangle \langle F_{z_1}^* \rangle + \lambda_2 \langle z_2 \rangle \langle F_{z_2}^* \rangle \right) \\ &= 2 \frac{M_{hid} \langle z_1 \rangle \langle z_2 \rangle}{M_{Pl}^4} e^{K/2M_{Pl}^2} \left(\lambda_1 |\langle z_1 \rangle|^2 + \lambda_2 |\langle z_2 \rangle|^2 \right) \simeq M_{hid} \end{aligned} \quad (20)$$

which lies at the desired scale. This solution for naturalness of the scale of the μ parameter[¶] is similar to the one proposed in [14]. The corresponding soft supersymmetry breaking Higgs bilinear mass-squared parameter reads as

$$B_H = \frac{1}{M_{Pl}^2} \left(\lambda_1 |\langle F_{z_1} \rangle|^2 + \lambda_2 |\langle F_{z_2} \rangle|^2 \right)$$

[¶]It might be instructive to contrast the model advocated here with low-scale MSSM extensions (by which we mean (i) the NMSSM which extends the MSSM with a chiral singlet superfield by forbidding a bare μ in the superpotential with a Z_3 symmetry, and (ii) the $U(1)'$ models which extend the MSSM by both a chiral singlet and an additional Abelian invariance) which also solve the μ problem in a dynamical fashion. The NMSSM suffers from the cosmological domain wall problem, and $U(1)'$ models spoil gauge coupling unification unless one introduces a number of exotic or family non-universal charge assignments. The present model is devoid of such problems as it utilizes global continuous symmetries operating at high scale.

$$= 4 \frac{|M_{hid} \langle z_1 \rangle \langle z_2 \rangle|^2}{M_{Pl}^6} e^{K/M_{Pl}^2} (\lambda_1 |\langle z_1 \rangle|^2 + \lambda_2 |\langle z_2 \rangle|^2) \simeq M_{hid}^2 \quad (21)$$

which is again at the right scale for keeping the MSSM Higgs sector sufficiently light.

The main implication of the model at hand is that μ parameter of the MSSM and right-handed neutrino masses are correlated with each other. It might be instructive to illustrate this correlation explicitly, and this is done in Fig. 1 by plotting μ and $m_{3/2}$ as functions of M_N for given values of λ' , λ_1 and λ_2 . We take $\lambda' \sim 10^{-4}$ so that for any value of M_N , $\langle z_1 \rangle [= (\lambda')^{-1} M_N]$ and in turn $\langle z_2 \rangle$ [see Eq. (11)] can be calculated. Then, as suggested by the figure, for $M_{hid} = 0.5$ TeV the μ parameter exhibits a strong dependence on M_N depending on the values of λ_1 and λ_2 . Indeed, as can be confirmed by using (20), the μ parameter remains around a TeV for $\lambda_1 = \lambda_2 = 1$ whereas it swings between the unphysical value zero and the desired value TeV when either λ_1 or λ_2 vanishes. The reason is that μ vanishes at zero $\langle z_1 \rangle$. In conclusion, the model offers a manifest correlation between μ and M_N , and μ gets properly stabilized to lie at a TeV when both λ_1 and λ_2 are nonvanishing.

For a detailed analysis of constraints on the model discussed in this section, it is necessary to confront it with the available laboratory, astrophysical and cosmological experimental data. In the next section we will provide a brief tour of the implications of the present model for a number of observables.

3 Phenomenological Implications

In this section we briefly discuss some phenomenologically interesting aspects of the model. The following remarks are in order:

- First of all, nonzero $\langle z_1 \rangle$ and $\langle z_2 \rangle$ lead to a spontaneous breakdown of the lepton number conservation and R invariance in the hidden sector. The spontaneous lepton number breaking releases a massless Goldstone boson, the Majoron J :

$$z_1 = (|\langle z_1 \rangle| + \phi_1) \times \exp i \left(\text{Arg} [\langle z_1 \rangle] + \frac{J}{|\langle z_1 \rangle|} \right) \quad (22)$$

where the real scalar fields ϕ_1 and J denote fluctuations about the vacuum state. In the flavor basis, the Majoron couples to right-handed neutrinos via $\lambda J N^T C N$; however, it

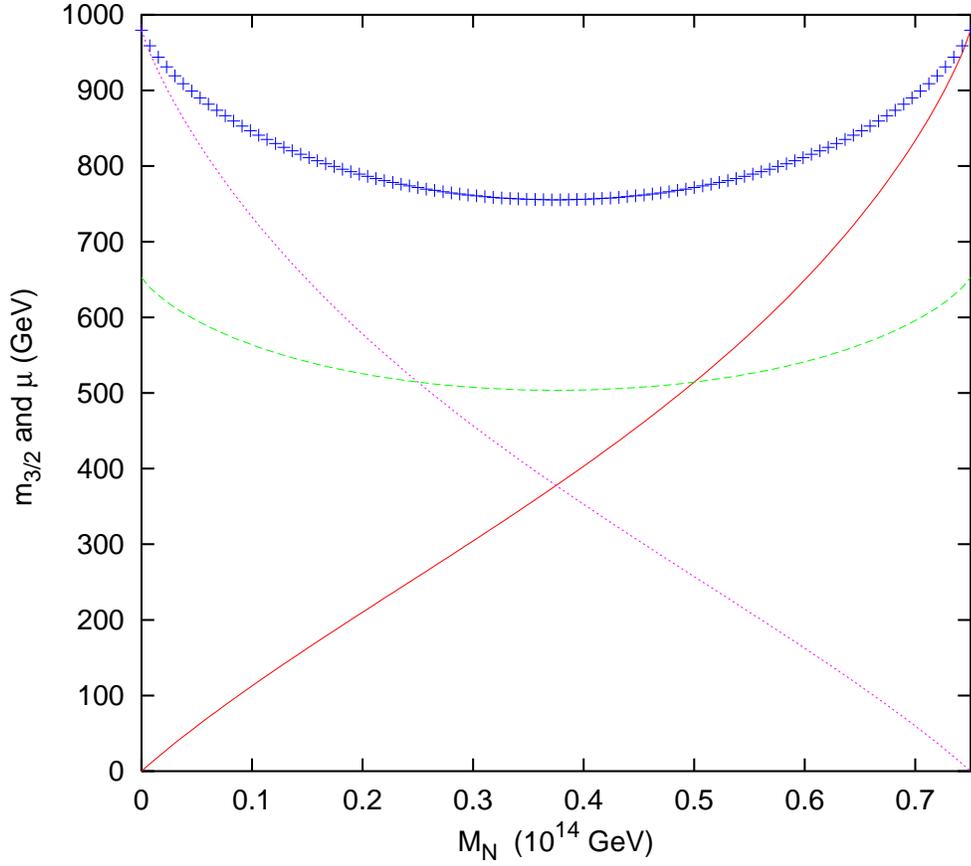


Figure 1: Variations of μ and $m_{3/2}$ with M_N for different values of the model parameters. We have set $\lambda' = \frac{10^{14} \text{ GeV}}{M_{Pl}}$ and $M_{hid} = 500 \text{ GeV}$. The solid and dotted lines show μ for $(\lambda_1 = 1, \lambda_2 = 0)$ and $(\lambda_1 = 0, \lambda_2 = 1)$, respectively. The curve depicted with crosses shows μ for $(\lambda_1 = \lambda_2 = 1)$. The dashed line stands for $m_{3/2}$.

does not couple to the left-handed active neutrinos. This means that in the mass basis, the light active neutrinos couple to the Majoron with an exceedingly small strength $\sim \lambda Y_\nu^2 \langle H_u \rangle^2 / M_N^2$. This coupling is too small to have any significance [20].

In this model, due to the spontaneous breakdown of R -symmetry there is an additional Goldstone boson, J' ,

$$z_2 = (|\langle z_2 \rangle| + \phi_2) \times \exp i \left(\text{Arg}[\langle z_2 \rangle] + \frac{J'}{|\langle z_2 \rangle|} \right) \quad (23)$$

where we have used a parametrization similar to (22). The coupling of J' to the SM particles is determined by the Kähler potential and is suppressed by $m_{SUSY}/M_{Pl} \sim 10^{-15} - 10^{-16}$ which is again too small to have any phenomenological consequences.

In the supersymmetric Majoron model infamous smajoron problem arises. In the following, we contrast the present model with supersymmetric Majoron model and compare status of smajoron problem in two scenarios. In the singlet Majoron model [21] there exists a new superfield \hat{S} which carries 2 units of lepton number and couples to the right-handed neutrinos via $W_1 = \hat{S} \hat{N}^c \hat{N}^c$. In similarity with the model proposed in this work, the right-handed neutrinos acquire masses through the VEV of \hat{S} . However, in this singlet Majoron model the mechanism responsible for nonvanishing VEVs is different from the one in our model: To develop VEVs additional superfields \hat{S}' and $\hat{\Lambda}$, with respective lepton numbers equal to -2 and 0, are introduced such that they interact through the superpotential $W_2 = \hat{\Lambda}(\hat{S}\hat{S}' - M^2)$. It can be shown that $\langle \hat{\Lambda} \rangle = 0$ while $\langle \hat{S} \rangle, \langle \hat{S}' \rangle \neq 0$. Since $\langle \hat{\Lambda} \rangle = 0$, one out of three linear combinations of the fermionic components of $\hat{\Lambda}$, \hat{S} and \hat{S}' is massless, and can be interpreted as Goldstino. However, in the present model, the mass matrix of ψ_{z_1} and ψ_{z_2} (the fermionic components of \hat{z}_1 and \hat{z}_2) is equal to

$$e^{\frac{K}{2M_{Pl}^2}} \begin{pmatrix} 0 & M_{hid} \\ M_{hid} & 0 \end{pmatrix} \quad (24)$$

and does not possess any vanishing eigenvalue. Namely, massless Majoron does not have any fermionic counterpart. This difference between the two models is not surprising at all since in the singlet Majoron model, supersymmetry is preserved [21]

($\langle F_S \rangle = \langle F_{S'} \rangle = \langle F_\Lambda \rangle = 0$) and a massless bosonic particle has to have a fermionic partner whereas in our model supersymmetry is broken ($\langle F_{z_1} \rangle, \langle F_{z_2} \rangle \neq 0$).

- Notice that spontaneous breakdowns of lepton number conservation and R invariance in the hidden sector reflect themselves as explicit breaking in the observable sector. Indeed, as is clear from $\widehat{W}_{obs-hid}$, $\langle z_1 \rangle \neq 0$ leads to explicit lepton number breaking via the induced right-handed neutrino mass. On the other hand, $\langle F_{z_{1,2}} \rangle \neq 0$ lead to spontaneous breakdown of supergravity whereby inducing explicit soft-breaking terms [13]. These soft terms lead to explicit breaking of R invariance down to its Z_2 subgroup, the R -parity, which is an exact discrete symmetry of the observable sector. Indeed, $\langle F_{z_1} \rangle$, for instance, induces a neutrino B-term which explicitly breaks the R invariance but conserves the R parity. As a result, the model accommodates a natural candidate for cold dark matter which is the famous lightest supersymmetric particle.
- In the present model, z_i and ψ_{z_i} ($i = 1, 2$) acquire masses of order of $m_{3/2}$. On the other hand, their couplings to MSSM spectrum are rather weak. Therefore, their decay rates are expected to be suppressed. For instance, their decay into Higgs fields occur with a rate

$$\Gamma(z_i \rightarrow HH) \sim \frac{M_{hid}}{4\pi} \left(\frac{M_{hid}}{M_{Pl}} \right)^2 \quad (25)$$

which is suppressed by the small ratio $\left(\frac{M_{hid}}{M_{Pl}} \right)^2 \sim 10^{-32}$. The smallness of \widehat{z}_i decay rates into SM species is rather generic.

At first sight, it may seem that this low but nonzero decay rate may be problematic for the big bang nucleosynthesis, especially in the face of the fact that at temperatures higher than M_N these particles can be produced via $NN \rightarrow zz$ at a rate not suppressed by M_{Pl} . Fortunately, a simple estimate shows that initial abundance of these particles cannot take too high values:

$$\begin{aligned} \frac{m_z n_z}{n_\gamma} &\sim m_z \Gamma(N \rightarrow zz) t \Big|_{T=M_N} \sim \left(\frac{|\lambda'|^4}{4\pi^3} T \right) \left(0.2 \frac{M_{Pl}}{\text{MeV}^2} \left(\frac{\text{MeV}}{T} \right)^2 \right) m_z \\ &\sim \left(\frac{m_z}{1000 \text{ GeV}} \right) \left(\frac{\lambda'}{10^{-4}} \right)^3 \times 10^{-12} \text{ GeV}. \quad (26) \end{aligned}$$

According to Fig. 3 of [25], the late decay of such particles cannot destroy the results of big bang nucleosynthesis.

- It might be instructive to analyze under what conditions one can make $\langle z_1 \rangle$ and $\langle z_2 \rangle$ hierarchically split so that $M_N \sim 10^{15}$ GeV arises with $\lambda' \sim \mathcal{O}(1)$. This can be achieved with a sufficiently small $1 + C_1$. Setting $C_1 = -1 + \epsilon^2$ with $\epsilon^2 \ll 1$, Eq. (11) implies

$$\begin{aligned} \langle z_1 \rangle &\sim \epsilon M_{Pl}, & \langle F_{z_1} \rangle &\sim \epsilon M_{Pl} M_{hid} \\ \langle z_2 \rangle &\sim M_{Pl}, & \langle F_{z_2} \rangle &\sim M_{Pl} M_{hid} \end{aligned} \quad (27)$$

which make it manifest that, for a sufficiently small ϵ , $\langle z_1 \rangle$ and $\langle F_{z_1} \rangle$ can be substantially smaller than, respectively, $\langle z_2 \rangle$ and $\langle F_{z_2} \rangle$. In fact, the VEVs in (27) suggest that

$$M_N = \lambda' \langle z_1 \rangle \sim \epsilon \lambda' M_{Pl}, \quad B_N = \lambda' \langle F_{z_1} \rangle \sim \lambda' \epsilon M_{Pl} M_{hid} \quad (28)$$

both of which involve ϵM_{Pl} rather than M_{Pl} itself. This ϵ dependence, however, is not present in the Higgs sector

$$\mu \simeq \lambda_2 \frac{\langle z_2 \rangle \langle F_{z_2} \rangle}{M_{Pl}^2} \sim M_{hid}, \quad B_H \simeq \lambda_2 \frac{|\langle F_{z_2} \rangle|^2}{M_{Pl}^2} \sim M_{hid}^2 \quad (29)$$

and $m_{3/2} \sim M_{hid}$ still holds. Finally, one notes that z_i and their fermionic counterparts weigh now M_{hid}/ϵ rather than M_{hid} . Taking $\epsilon \sim 10^{-3}$, the existing neutrino data can be explained with $\lambda' \sim 1$.

- Let us analyze the Higgs sector in more detail. Note that, without loss of generality, we can absorb the phase of M_{hid} by rephasing \widehat{z}_2 . Therefore, hereon we are going to assume that M_{hid} is real. First consider the phase of the μ parameter. From (21) and (20) it follows that the relative phase between μ and B_H is determined solely by the total phase of $\langle z_1 \rangle \langle z_2 \rangle$. More explicitly,

$$\text{Arg} \left[\frac{\mu}{B_H} \right] = \text{Arg} [\langle z_1 \rangle] + \text{Arg} [\langle z_2 \rangle] \quad (30)$$

as follows from (22) and (23). This is a physical basis-independent phase that cannot be rotated away by redefinition of the relative phase between \widehat{H}_u and \widehat{H}_d . In the context

of both constrained [24] and unconstrained [26] MSSM, the present bounds on electric dipole moments imply that $\text{Arg}[\langle z_1 \rangle \langle z_2 \rangle]$ cannot exceed a few percent. Therefore, the VEVs $\langle z_1 \rangle$ and $\langle z_2 \rangle$ must be nearly back-to-back in the vacuum of the theory.

The structure of the model entails certain correlations among certain parameters which would bear no correlation in the MSSM, constrained or otherwise. An interesting case concerns the phase of the μ parameter. To see this, consider the Dirac coupling of sneutrinos in the soft-breaking sector:

$$\mathcal{L} \ni a_0 Y'_\nu \tilde{L} \cdot H_u \widetilde{N}^c + \text{h.c.} \quad (31)$$

which respects both the lepton number and R -parity. Here a_0 is the universal trilinear coupling, as appropriate for the constrained MSSM. It can be shown that at two-loop level this operator induces a bilinear interaction between z_1 and z_2 as follows

$$\mathcal{L} \ni c_z^* M_{hid} z_1 z_2 + \text{h.c.} \quad (32)$$

where

$$c_z^* = (\text{an } \mathcal{O}(1) \text{ real number}) \times a_0 \frac{\lambda'^T Y'_\nu{}^* Y_\nu'^T \lambda'^*}{(16\pi^2)^2} \quad (33)$$

which is a small perturbation in size. However, an interaction of the form (32) leads to direct alignment of $\langle z_1 \rangle \langle z_2 \rangle$ towards c_z for scalar potential (in terms of $H_u^0, H_d^0, \tilde{\nu}_L^i, \widetilde{N}^c_i$) to possess a local minimum. Therefore, the phase of μ (*i.e.* $\text{Arg}[\langle z_1 \rangle \langle z_2 \rangle]$) relaxes to that of c_z and hence to the phase of a_0 according to (33). (Note that the combination $\lambda'^T Y'_\nu{}^* Y_\nu'^T \lambda'^*/(16\pi^2)^2$ is real.) Consequently, the phase of μ gets traded for that of the trilinear couplings, and as a by-product of this correlation, in future if a nonzero d_e is measured, within this model, we can extract the phase of μ which corresponds to the phase of a_0 and then, in principle, we can predict the values d_{Hg} and d_n and test these predictions in the laboratory experiments [27].

Having discussed the CP-odd phase of the μ parameter, we now analyze the role of the B_H parameter in some depth. In general, μ parameter can be determined by measuring masses and mixing angles of the charginos and neutralinos. On the other hand by studying the mass and decay modes of the CP-odd Higgs boson, A_0 , we can

derive the value of B_H [28]. Therefore, the ratio $|B_H/\mu|$ can be measured in a rather model-independent way. It is straightforward to show that

$$2M_{hid} < |B_H/\mu| = 2m_{3/2} < 2.6M_{hid} \quad (34)$$

in the present model. Since interactions of gravitino are suppressed by M_{Pl}^{-1} , it cannot, in practice, be detected at colliders. However, gravitino mass affects cosmological observations, opening a window for testing this relation. Indeed, from the relation

$$|\mu|^2 + m_{H_d}^2 = B_H \tan \beta - \frac{m_Z^2}{2} \cos 2\beta \quad (35)$$

we expect $|B_H/\mu| \sim |\mu|/\tan \beta$ which means that at large $\tan \beta$, there is a “little hierarchy” among the parameters of the model. Within our model this implies that gravitino, rather than the lightest neutralino, with mass $\sim |\mu|/\tan \beta$, might be the lightest supersymmetric particle and hence a candidate for cold dark matter (see, for instance, [29] for gravitino dark matter in constrained MSSM).

The discussions above provide a brief summary of the implications of the model constructed in Sec. 2. For a proper description of the phenomenology of this model, it is necessary to perform a detailed analysis of various quantities of phenomenological interest.

4 Conclusion

In this work we have constructed a hidden sector model, within $N = 1$ supergravity, for generating, upon supergravity breakdown, the μ parameter of the MSSM and the right-handed neutrino mass M_N at their right scales. The model utilizes global lepton number conservation and R invariance to forbid bare μ and M_N parameters appearing in the superpotential. Moreover, the model employs a non-minimal Kähler potential exhibiting logarithmic dependencies on the hidden sector fields. The origin of these non-minimal contributions are left unexplained, yet string compactification has been an inspiring source. We have determined parameter regions where M_N and μ come out to lie at their right scales, and found that the VEVs of the hidden sector fields can exhibit a hierarchical splitting so as to reduce unnatural tunings of the superpotential parameters. As footnoted in the text, the model at

hand neither suffers from domain wall problem, nor exhibits any tension with gauge coupling unification as encountered, respectively, in NMSSM and $U(1)'$ models.

We have confronted the model put forward with a number of observables, and identified distinctive features and ways of evading the existing bounds from various sources. The model predicts $|B_H/\mu| = 2m_{3/2}$ which means, for a large part of parameter space, the gravitino is the lightest supersymmetric particle and thus a candidate for dark matter. Consequently, the mechanism advocated in this work possesses potential implications for various observables, a global analysis of which is yet to be performed.

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References

- [1] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
- [2] S. K. Soni and H. A. Weldon, Phys. Lett. B **126**, 215 (1983).
- [3] E. Witten, Nucl. Phys. B **188**, 513 (1981); I. Affleck, M. Dine and N. Seiberg, Phys. Rev. Lett. **52**, 1677 (1984).
- [4] A. Yu. Smirnov, arXiv:hep-ph/0411194.
- [5] M. Gell-Mann, P. Ramond and R. Slansky, Print-80-0576 (CERN) (see also: P. Ramond, arXiv:hep-ph/9809459.) T. Yanagida, *In Proceedings of the Workshop on the*

- Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979*; S. L. Glashow, HUTP-79-A059 *Based on lectures given at Cargese Summer Inst., Cargese, France, Jul 9-29, 1979*; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.
- [6] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B **104**, 199 (1981); A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980) [Yad. Fiz. **31**, 497 (1980)]; J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).
- [7] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [8] W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. **315**, 305 (2005) [arXiv:hep-ph/0401240]; G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, "MSSM," Nucl. Phys. B **685**, 89 (2004) [arXiv:hep-ph/0310123]; see however, Y. Farzan and J. W. F. Valle, arXiv:hep-ph/0509280.
- [9] E. J. Chun, J. E. Kim and H. P. Nilles, Nucl. Phys. B **370** (1992) 105.
- [10] The LEP SUSY working group, <http://lepsus.web.cern.ch/lepsusy/>.
- [11] G. Belanger, F. Boudjema, A. Cottrant, A. Pukhov and S. Rosier-Lees, JHEP **0403**, 012 (2004) [arXiv:hep-ph/0310037].
- [12] P. Fayet, Phys. Lett. B **69**, 489 (1977). J. E. Kim and H. P. Nilles, Phys. Lett. B **138**, 150 (1984); D. Suematsu and Y. Yamagishi, Int. J. Mod. Phys. A **10**, 4521 (1995) [arXiv:hep-ph/9411239]; M. Cvetič and P. Langacker, Mod. Phys. Lett. A **11**, 1247 (1996) [arXiv:hep-ph/9602424]; V. Jain and R. Shrock, arXiv:hep-ph/9507238; Y. Nir, Phys. Lett. B **354**, 107 (1995) [arXiv:hep-ph/9504312].
- [13] S. Dimopoulos and S. D. Thomas, Nucl. Phys. B **465**, 23 (1996) [arXiv:hep-ph/9510220]; D. A. Demir, Phys. Rev. D **62**, 075003 (2000) [arXiv:hep-ph/9911435].
- [14] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

- [15] L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B **329**, 27 (1990); J. P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B **372**, 145 (1992).
- [16] M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992) [arXiv:hep-th/9202003]; R. Kallosh, A. D. Linde, D. A. Linde and L. Susskind, Phys. Rev. D **52**, 912 (1995) [arXiv:hep-th/9502069]; R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins and L. M. Widrow, Phys. Lett. B **282**, 132 (1992) [arXiv:hep-ph/9203206]; S. M. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992)
- [17] R. Kallosh, A. D. Linde, D. A. Linde and L. Susskind, Phys. Rev. D **52**, 912 (1995) [arXiv:hep-th/9502069].
- [18] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. **91**, 251801 (2003) [arXiv:hep-ph/0307081]; G. D'Ambrosio, G. F. Giudice and M. Raidal, Phys. Lett. B **575**, 75 (2003) [arXiv:hep-ph/0308031].
- [19] Y. Farzan, JHEP **0502**, 025 (2005) [arXiv:hep-ph/0411358]; Y. Farzan, Phys. Rev. D **69**, 073009 (2004) [arXiv:hep-ph/0310055]; Y. Farzan, arXiv:hep-ph/0505004.
- [20] G. F. Giudice, A. Masiero, M. Pietroni and A. Riotto, Nucl. Phys. B **396**, 243 (1993) [arXiv:hep-ph/9209296]; Y. Farzan, Phys. Rev. D **67**, 073015 (2003) [arXiv:hep-ph/0211375].
- [21] R. N. Mohapatra and X. Zhang, Phys. Rev. D **49**, 1163 (1994) [Erratum-ibid. D **49**, 6246 (1994)] [arXiv:hep-ph/9307231].
- [22] M. Bolz, W. Buchmuller and M. Plumacher, Phys. Lett. B **443**, 209 (1998) [arXiv:hep-ph/9809381]; M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B **606**, 518 (2001) [arXiv:hep-ph/0012052].
- [23] W. Buchmuller, R. D. Peccei and T. Yanagida, arXiv:hep-ph/0502169.
- [24] T. Ibrahim and P. Nath, Phys. Rev. D **58**, 111301 (1998) [Erratum-ibid. D **60**, 099902 (1999)] [arXiv:hep-ph/9807501]; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B **680**, 339 (2004) [arXiv:hep-ph/0311314].

- [25] J. R. Ellis, G. B. Gelmini, J. L. Lopez, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **373**, 399 (1992). See, however, R. Allahverdi and A. Mazumdar, arXiv:hep-ph/0512227; arXiv:hep-ph/0505050.
- [26] M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D **59**, 115004 (1999) [arXiv:hep-ph/9810457]; S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B **606** (2001) 151 [arXiv:hep-ph/0103320];
- [27] D. A. Demir and Y. Farzan, JHEP **0510**, 068 (2005) [arXiv:hep-ph/0508236].
- [28] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, “The Higgs Hunter’s Guide,” SCIPP-89/13, (1989)
- [29] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B **588**, 7 (2004) [arXiv:hep-ph/0312262].