

Generalized Modified Gravity in Large Extra Dimensions

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Abstract

We discuss effective interactions among brane matter induced by modifications of higher dimensional Einstein gravity through the replacement of Einstein-Hilbert term with a generic function $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ of the curvature tensors. We determine gravi-particle spectrum of the theory, and perform a comparative analysis of its predictions with those of the Einstein gravity within Arkani-Hamed–Dvali–Dimopoulos (ADD) setup. We find that this general higher-curvature quantum gravity theory contributes to scatterings among both massive and massless brane matter (in contrast to much simpler generalization of the Einstein gravity, $f(\mathcal{R})$, which influences only the massive matter), and therefore, can be probed via various scattering processes at present and future colliders and directly confronted with the ADD expectations. In addition to collision processes which proceed with tree-level gravi-particle exchange, effective interactions among brane matter are found to exhibit a strong sensitivity to higher-curvature gravity via the gravi-particle loops. Furthermore, particle collisions with missing energy in their final states are found to be sensitive to additional gravi-particles not found in Einstein gravity. In general, road to a correct description of quantum gravity above Fermi energies depends crucially on if collider and other search methods end up with a negative or positive answer for the presence of higher-curvature gravitational interactions.

1 Introduction

The extra spatial dimensions (large [1], warped [2] or hyperbolic [3]) have proven useful in solving the gauge hierarchy problem within the quantum gravitational framework. In particular, large extra dimensions induce Newton's constant in four dimensions from TeV scale Einstein gravity via the large volume of the extra space. The basic setup of this scenario *i.e.* Arkani-Hamed–Dimopoulos–Dvali (ADD) scenario [1], is that $(1 + 3)$ -dimensional universe we live in is a field-theoretic brane [4] which traps all flavors of matter except the SM singlets *e.g.* the graviton and right-handed neutrinos. As long as the surface tension of the brane does not exceed the fundamental scale \overline{M}_D of D -dimensional gravity, at distances $\gg 1/\overline{M}_D$ the spacetime metric g_{AB} remains essentially flat. In other words, for singlet emissions (from brane) with transverse (to brane) momenta $|\vec{p}_T| \ll \overline{M}_D$ the background spacetime is basically Minkowski. Therefore, it is admissible to expand D -dimensional metric about the flat background

$$g_{AB} = \eta_{AB} + 2\overline{M}_D^{1-D/2} h_{AB} \quad (1)$$

where $\eta_{AB} = \text{diag.}(1, -1, -1, \dots, -1)$ and h_{AB} are perturbations. The gravitational sector is described by the Einstein-Hilbert action

$$S_{ADD} = \int d^D x \sqrt{-g} \left\{ -\frac{1}{2} \overline{M}_D^{D-2} \mathcal{R}(g_{AB}) + \mathcal{L}_{matter}(g_{AB}, \psi) \right\} \quad (2)$$

where ψ collectively denotes matter fields localized on the brane. There are various ways [1] to see that the Planck scale seen on the brane is related to the fundamental scale of gravity in higher dimensions via

$$\overline{M}_{Pl} = \sqrt{V_\delta} \overline{M}_D^{1+\delta/2} \quad (3)$$

which equals $(2\pi R)^{1/2} \overline{M}_D^{1+\delta/2}$ when $\delta \equiv D - 4$ extra spatial dimensions are compactified over a torus of radius R . Obviously, larger the R closer the \overline{M}_D to the electroweak scale [1]. Upon compactification, the higher dimensional graviton gives rise to a tower of massive S, P and D states on the brane, and they participate in various scattering processes involving radiative corrections to SM parameters, missing energy signals as well as graviton exchange

processes. The collider signatures of these processes have been discussed in detail in seminal papers [5, 6].

The ADD mechanism is based on higher dimensional Einstein gravity with the metric (1). However, given general covariance alone, there is no symmetry reason to guarantee that the action density in (2) is unique. Indeed, general covariance does not forbid the action density in (2) to be generalized to a generic function $f(\mathcal{R}, \square\mathcal{R}, \nabla_A\mathcal{R}\nabla^A\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD}, \dots)$ of curvature invariants. In fact, such modifications of Einstein gravity have already been proposed and utilized for purposes of improving the renormalizability of the theory [7, 8] and for explaining recent acceleration of the universe [9, 10]. Of course, once we depart from the minimal Einstein-Hilbert regime there is no rule whatsoever which can limit numbers and types of the invariants. Our approach here, however, is to consider only those invariants which are of lowest mass dimension and are quadratic contractions of the curvature tensors: $\mathcal{R}_{AB}\mathcal{R}^{AB}$ and $\mathcal{R}_{ABCD}\mathcal{R}^{ABCD}$ in spite of fact that we do not have any symmetry reason for not considering the higher-derivative ones $\square\mathcal{R}, \nabla_A\mathcal{R}\nabla^A\mathcal{R}, \nabla_C\mathcal{R}_{AB}\nabla^C\mathcal{R}^{AB}$, etc. In effect, we generalize Einstein-Hilbert term to a generic function $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ of the curvature invariants, and derive and analyze effective interactions among brane matter induced by such modifications of higher dimensional Einstein gravity.

The simplest generalization of (2) would be to consider a generic function $f(\mathcal{R})$ of the curvature scalar. This possibility has been analyzed in detail in the recent work [11], and it has been found that $f(\mathcal{R})$ gravity effects are particularly pronounced and become distinguishable from those of the Einstein gravity in scattering processes involving massive brane matter *i.e.* heavy fermions, weak bosons and the Higgs boson (for recent work on Lovelock gravity see [12]). The reason is that $f(\mathcal{R})$ theory is equivalent to Einstein gravity plus an independent scalar field theory, and it is the propagation of this additional scalar that causes observable differences between the $f(\mathcal{R})$ gravity and ADD setup in high energy processes [11].

Here it is worth emphasizing that considering $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ theory instead of $f(\mathcal{R})$ gravity is not a straightforward generalization. The reason is that the former is a four-derivative theory, and it is generically endowed with a spin=2 ghost [8]. The presence of such negative-norm states constitutes the main difference between the two types

of modified gravity theories, and goal of the present work is to examine their signatures in high-energy processes in a comparative fashion. Such ghostly states are obviously dangerous in four dimensions (care should be paid to nonlinearities though) especially at large distances [9, 10]; however, in a higher dimensional setting, it is the experiment (at the LHC or ILC) which will eventually establish presence or absence of such states whereby providing a deeper understanding of yet-to-be found quantum theory of gravity.

In this work we will consider a general modification of the Einstein gravity, and discuss its physics implications in comparison with the ADD and $f(\mathcal{R})$ gravity setups. In Sec. 2 below we derive graviton propagator and describe how it interacts with brane matter. Here we put special emphasis on virtual graviton exchange. In Sec. 3 we study a number of higher dimensional operators which are sensitive to modified gravity effects. In Sec. 4 we briefly discuss some further signatures of modified gravity concerning graviton production and decay as well as certain loop observables on the brane. In Sec. 5 we conclude.

2 Virtual Gravi-Particle Exchange

The modification of the Einstein gravity we consider is parameterized by

$$S = \int d^D x \sqrt{-g} \left\{ -\frac{1}{2} M_D^{D-2} f(\mathcal{R}, P, Q) + \mathcal{L}_{matter}(g_{AB}, \psi) \right\} \quad (4)$$

where couplings to matter fields ψ are identical to those in (2). Here P and Q , as in [9, 10], stand, respectively, for the quadratic contractions of the Ricci and Riemann tensors:

$$P = \mathcal{R}_{AB} \mathcal{R}^{AB}, \quad Q = \mathcal{R}_{ABCD} \mathcal{R}^{ABCD} \quad (5)$$

which contain four derivatives. The metric field obeys

$$\begin{aligned} & [\nabla_A \nabla_B - g_{AB} \square - \mathcal{R}_{AB}] f_R \\ & + [2\nabla_A \nabla^C \mathcal{R}_{CB} - \square \mathcal{R}_{AB} - g_{AB} \nabla^C \nabla^D \mathcal{R}_{CD} - 2\mathcal{R}_{CA} \mathcal{R}_B^C] f_P \\ & + [4\nabla^C \nabla^D \mathcal{R}_{CBAD} - 2\mathcal{R}_{CDEA} \mathcal{R}_B^{CDE}] f_Q + \frac{1}{2} f g_{AB} = \frac{\mathcal{T}_{AB}}{M_D^{D-2}} \end{aligned} \quad (6)$$

where $f_R \equiv \partial f / \partial R$, $f_P \equiv \partial f / \partial P$, $f_Q \equiv \partial f / \partial Q$, and

$$\mathcal{T}_{AB} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_{matter})}{\delta g^{AB}} = \delta^\delta(\vec{y}) \delta_A^\mu \delta_B^\nu T_{\mu\nu}(z) \quad (7)$$

is the stress tensor of the brane matter where y_i and z_μ stand, respectively, for coordinates in extra space and on the brane. The second equality here reflects the fact that entire energy and momentum are localized on the brane. Clearly, energy-momentum flow has to be conserved $\nabla^A \mathcal{T}_{AB} = 0$, and this is guaranteed to happen provided that $\nabla^\mu T_{\mu\nu} = 0$.

Obviously, the equations of motion (6) reduce to Einstein equations when $f(\mathcal{R}, P, Q) = \mathcal{R}$. In general, for analyzing dynamics of small oscillations about a background geometry, $g_{AB} = g_{AB}^0$ with curvature scalar \mathcal{R}_0 , $f(\mathcal{R}, P, Q)$ must be regular at $\mathcal{R} = \mathcal{R}_0$. In particular, as suggested by (6), $f(\mathcal{R}, P, Q)$ must be regular at the origin and $f(0, 0, 0)$ must vanish (*i.e.* bulk cosmological constant must vanish) for $f(\mathcal{R}, P, Q)$ to admit a flat background geometry.

For determining how higher curvature gravity (4) influences interactions among the brane matter, it is necessary to determine the propagating modes which couple to the matter stress tensor. This requires expansion of the action density by using (1) up to the desired order in h_{AB} . The zeroth order term obviously vanishes. The terms first order in h_{AB} vanish by the equations of motion (6). The quadratic part, on the other hand, turns out to be

$$S_h = \int d^D x \left[\frac{1}{2} h_{AB}(x) \mathcal{O}^{ABCD}(x) h_{CD}(x) - \frac{1}{M_D^{(D-2)/2}} h_{AB}(x) \mathcal{T}(x)^{AB} \right] \quad (8)$$

such that propagator of $h_{AB}(x)$, defined via the relation

$$\mathcal{O}_{ABCD}(x) \mathcal{D}^{CDEF}(x, x') = \frac{1}{2} \delta^D(x - x') \left(\delta_A^E \delta_B^F + \delta_B^E \delta_A^F \right), \quad (9)$$

takes the form

$$\begin{aligned} -i \mathcal{D}^{ABCD}(p^2) &= d_1(p^2) \eta^{AB} \eta^{CD} + d_2(p^2) \left(\eta^{AC} \eta^{BD} + \eta^{AD} \eta^{BC} \right) \\ &+ d_3(p^2) \left(p^A p^B \eta^{CD} + \eta^{AB} p^C p^D \right) \\ &+ d_4(p^2) \left(\eta^{BC} p^A p^D + \eta^{AD} p^B p^C + \eta^{AC} p^B p^D + \eta^{BD} p^A p^C \right) \\ &+ d_5(p^2) p^A p^B p^C p^D \end{aligned} \quad (10)$$

where the form factors $d_{1,\dots,5}(p^2)$ depend on the underlying theory of gravitation. In ADD setup, based on Einstein gravity, they are given by $d_1(p^2) = -1/(D-2)p^2$, $d_2(p^2) = 1/2p^2$, $d_4(p^2) = (\xi - 1)/2p^4$, $d_3(p^2) = d_5(p^2) = 0$. In $f(\mathcal{R})$ gravity none of them vanishes and their explicit expressions can be found in [11]. In the framework of modified gravity discussed here,

they obtain nontrivial structures, too. For graviton-mediated interactions among brane-localized matter with conserved energy-momentum, only $d_1(p^2)$ and $d_2(p^2)$ are relevant, and they are given by

$$\begin{aligned} d_1(p^2) &= -\frac{1}{(D-2)f_R(0)p^2} + \frac{1}{(D-1)f_R(0)(p^2 - m_1^2)} + \frac{1}{(D-1)(D-2)f_R(0)(p^2 - m_\phi^2)} \\ d_2(p^2) &= \frac{1}{2f_R(0)p^2} - \frac{1}{2f_R(0)(p^2 - m_1^2)} \end{aligned} \quad (11)$$

where we introduced the mass scales

$$m_0^2 = -\frac{4f_R(0)}{f_P(0) - 8f_{RR}(0)}, \quad m_1^2 = -\frac{4f_R(0)}{f_P(0) + 4f_Q(0)}, \quad m_\phi^2 = -\frac{(D-2)m_0^2 m_1^2}{(D-1)m_1^2 + m_0^2} \quad (12)$$

parameterizing the non-minimal nature of the gravity theory considered. The remaining form factors $d_{3,4,5}(p^2)$ can be obtained from (9) straightforwardly. Though it does not appear in $d_1(p^2)$ and $d_2(p^2)$ above, in general, the propagator depends on the gauge-fixing parameter ξ following from the gauge-fixing term

$$\mathcal{L}_g = \frac{f_R(0)}{\xi} \eta^{AC} \left(\partial^B h_{AB} - \frac{1}{2} \partial_A h_B^B \right) \left(\partial^D h_{CD} - \frac{1}{2} \partial_C h_D^D \right) \quad (13)$$

added to the action density in (8). Here, $f_R(0)$ is introduced to match the terms generated by \mathcal{L}_g with the ones in (8). The de Donder gauge, $\xi = 1$, is frequently employed in quantum gravity.

Having determined the propagator, it is timely to analyze gravi-particles in the system and their propagation characteristics. The propagating modes and their properties are determined by the pole structures of $d_1(p^2)$ and $d_2(p^2)$ (and by the remaining form factors $d_{3,4,5}(p^2)$ when the longitudinal polarizations are taken into account). Indeed, the pole at $p^2 = 0$ guarantees the existence of a massless $J = 2$ mode in D dimensions. That this is the case directly follows from the projector

$$\frac{1}{2} \left(\eta^{AC} \eta^{BD} + \eta^{AD} \eta^{BC} \right) - \frac{1}{D-2} \eta^{AB} \eta^{CD} \quad (14)$$

multiplying $1/p^2$. By restoring the longitudinal components via the replacement $\eta_{AB} \rightarrow \eta_{AB} - p_A p_B / p^2$ in each term of (14) one ensures that the normal mode under concern corresponds to a massless $J = 2$ excitation, the graviton [8]. In fact, when $f_R(0) = 1$, $f_Q(0) = 0$,

$f_{RR}(0) = 0$ and $f_P(0) = 0$ the whole propagator (10), as it should, reduces to that computed in the Einstein gravity [5, 6].

The second terms of $d_1(p^2)$ and $d_2(p^2)$ combine to give a massive $J = 2$ propagating mode. Indeed, these two terms result in weighing of $1/(p^2 - m_1^2)$ with the projector

$$\frac{1}{2} \left(\eta^{AC} \eta^{BD} + \eta^{AD} \eta^{BC} \right) - \frac{1}{D-1} \eta^{AB} \eta^{CD} \quad (15)$$

which corresponds to a massive $J = 2$ excitation in D dimensions. The most spectacular aspect of this propagator is that it has a negative residue that is the excitation under concern is a ghost represented by negative norm states in Hilbert space. This can be cured by no choice of the model parameters because the sole and obvious choice of negative $f_R(0)$ converts the massless graviton discussed above into a ghost – an absolutely unwanted situation since then theory possesses no Einsteinian limit at any mass scale. The existence of this tensorial ghost is a characteristic property of higher curvature gravity consisting of Ricci and Riemann tensors [8], and it actually plays a rather affirmative role in cancelling the divergences in loop calculations in the same sense as the Pauli-Villars regulation does in quantum field theory.

In addition to the aforementioned tensor modes, as evidenced by the second line of $d_1(p^2)$, the particle spectrum also consists of a scalar particle with mass-squared m_ϕ^2 . Indeed $1/(p^2 - m_\phi^2)$ is weighted by the projector

$$\eta^{AB} \eta^{CD} \quad (16)$$

which guarantees the scalar nature of the propagating mode. This mode is a tachyon as long as m_0^2 and m_1^2 have the same sign otherwise it is a true scalar field. The parameter values *i.e.* signs of m_1^2 and m_0^2 , competition between them as well as various other factors give rise to several possibilities. An interesting limit concerns $m_1^2 \rightarrow \pm\infty$, which can be achieved by taking a special $f(\mathcal{R}, P, Q)$ with $f_P(0) = -4f_Q(0)$ or $f_P(0) = 0 = f_Q(0)$, then the tensor ghost completely decouples from the spectrum. However, the scalar field continues to accompany the tensor ghost with mass $m_\phi^2 = -((D - 2)/(D - 1))m_0^2$ in agreement with [11]. For generating the pure Einstein gravity one needs to send both m_0^2 and m_1^2 to ∞ which necessitates $f_P(0), f_Q(0), f_{RR}(0) \rightarrow 0$.

Having determined the gravi-particle spectrum of $f(\mathcal{R}, P, Q)$ gravity in D dimensions, we start analyzing the consequences of the compactness of the extra space. Indeed, by letting

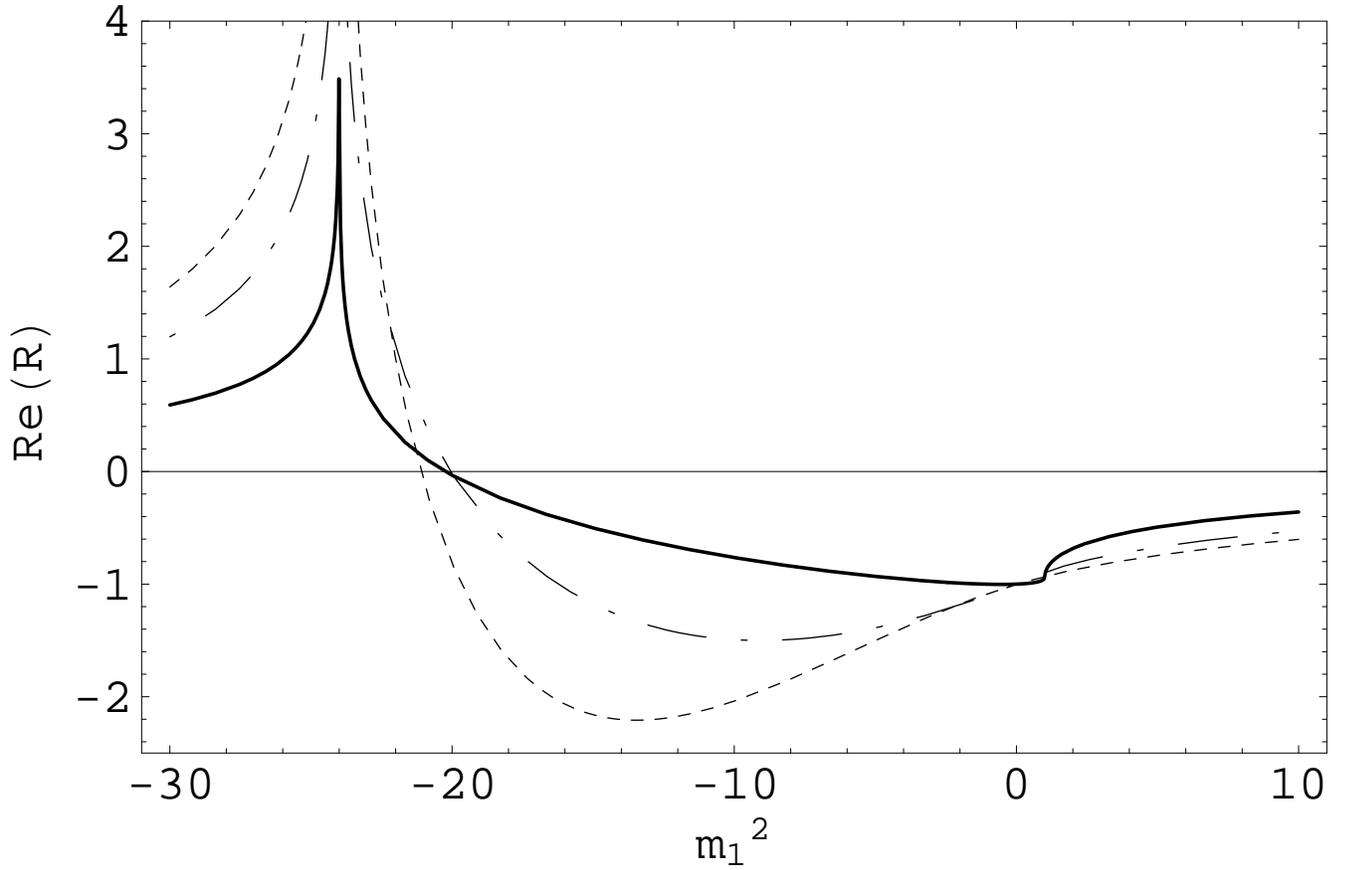


Figure 1: The dependence of $\text{Re}[R(k^2)]$ on m_1^2 for $k^2 = (1 \text{ TeV})^2$, $\Lambda = \overline{M}_D = 5 \text{ TeV}$, and $\delta = 3$ (solid curve), $\delta = 5$ (dot-dashed curve) and $\delta = 7$ (short-dashed curve). In the plot m_1^2 varies from -30 TeV^2 up to $+10 \text{ TeV}^2$.

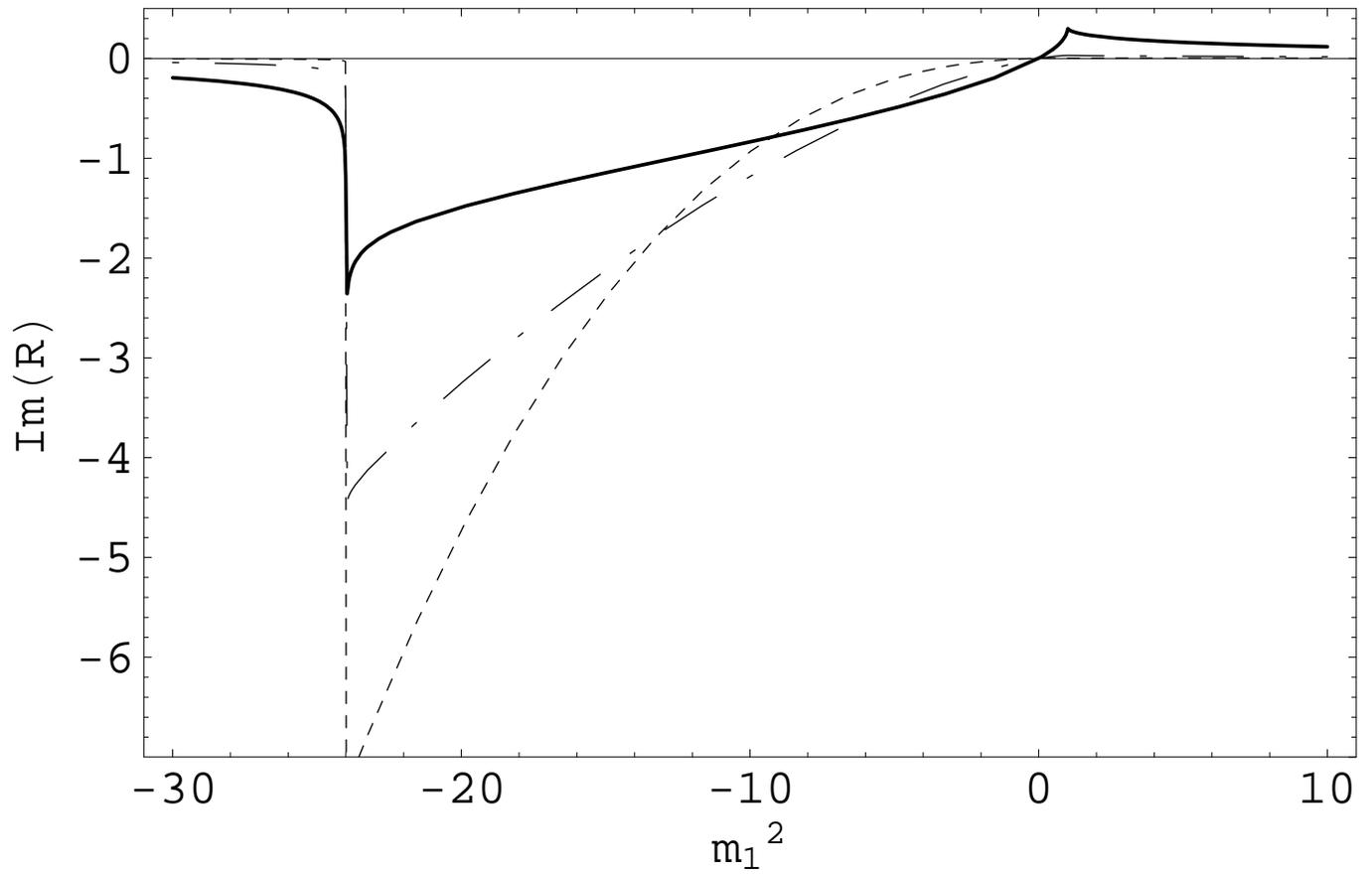


Figure 2: The same as in Fig. 1 but for $\text{Im}[R(k^2)]$.

extra space be torus-shaped with radius R as in the ADD mechanism, the matter stress tensor obeys the Kaluza-Klein expansion

$$\mathcal{T}_{AB}(x) = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{\sqrt{V_\delta}} e^{-i(k \cdot z - \frac{\vec{n} \cdot \vec{y}}{R})} \delta_A^\mu \delta_B^\nu T_{\mu\nu}(k) \quad (17)$$

where (n_1, \dots, n_δ) is a δ -tuple of integers. Given this Fourier decomposition of the stress tensor, the amplitude for an on-brane system a to make a transition into another on-brane system b becomes

$$\mathcal{A}(k^2) = \frac{1}{\overline{M}_{Pl}^2} \sum_{\vec{n}} T_{\mu\nu}^{(a)}(k) \mathcal{D}^{\mu\nu\lambda\rho} \left(k^2 - \frac{\vec{n} \cdot \vec{n}}{R^2} \right) T_{\lambda\rho}^{(b)}(k) \quad (18)$$

where use has been made of (3) in obtaining $1/\overline{M}_{Pl}^2$ factor in front. Though we are dealing with a tree-level process the amplitude involves a summation over all Kaluza-Klein levels due to the fact that these states are inherently virtual because of their propagation off the brane. Conservation of energy and momentum implies that only the first two terms in the propagator (10) contributes to (18), and after performing summation the transition amplitude takes the form

$$\begin{aligned} \mathcal{A}(k^2) = & \frac{S_{\delta-1}}{(2\pi)^\delta} \frac{1}{\overline{M}_D^4 f_R(0)} \left(\frac{\Lambda}{\overline{M}_D} \right)^{\delta-2} \left\{ \right. \\ & \mathcal{G} \left(\frac{\Lambda}{\sqrt{k^2}} \right) \left(T_{\mu\nu}^{(a)} T^{(b)\mu\nu} - \frac{1}{\delta+2} T_\mu^{(a)\mu} T_\nu^{(b)\nu} \right) \\ & - \mathcal{G} \left(\frac{\Lambda}{\sqrt{k^2 - m_1^2}} \right) \left(T_{\mu\nu}^{(a)} T^{(b)\mu\nu} - \frac{1}{\delta+3} T_\mu^{(a)\mu} T_\nu^{(b)\nu} \right) \\ & \left. + \frac{1}{(\delta+2)(\delta+3)} \mathcal{G} \left(\frac{\Lambda}{\sqrt{k^2 - m_\phi^2}} \right) T_\mu^{(a)\mu} T_\nu^{(b)\nu} \right\} \quad (19) \end{aligned}$$

which exhibits a huge overall enhancement $\mathcal{O}(\overline{M}_{Pl}^2/\overline{M}_D^2)$ compared to (18) due to the contributions of finely-spaced Kaluza-Klein levels [1]. Here $S_{\delta-1} = (2\pi^{\delta/2})/\Gamma(\delta/2)$ is the surface area of δ -dimensional unit sphere and Λ (which is expected to be $\mathcal{O}(\overline{M}_D)$ since above \overline{M}_D underlying quantum theory of gravity completes the classical treatment pursued here) is the ultraviolet cutoff needed to tame divergent summation over Kaluza-Klein levels. In fact,

$\mathcal{A}(q^2)$ exhibits a strong dependence on Λ , as suggested by (see also the corresponding series expressions derived in [5, 6])

$$\mathcal{G}\left(\frac{\Lambda}{\sqrt{q^2}}\right) = -i\frac{\pi}{2}\left(\frac{q^2}{\Lambda^2}\right)^{\frac{\delta}{2}-1} + \frac{\pi}{2}\left(\frac{q^2}{\Lambda^2}\right)^{\frac{\delta}{2}-1} \cot\frac{\pi\delta}{2} - \frac{1}{\delta-2} {}_2F_1\left(1, 1 - \frac{\delta}{2}, 2 - \frac{\delta}{2}, \frac{q^2}{\Lambda^2}\right) \quad (20)$$

for $0 \leq q^2 \leq \Lambda^2$, and

$$\mathcal{G}\left(\frac{\Lambda}{\sqrt{q^2}}\right) = \frac{1}{\delta} \frac{\Lambda^2}{q^2} {}_2F_1\left(1, \frac{\delta}{2}, 1 + \frac{\delta}{2}, \frac{\Lambda^2}{q^2}\right) \quad (21)$$

for $q^2 < 0$ or $q^2 > \Lambda^2$. The imaginary part of \mathcal{G} , relevant for the timelike propagator (20), is generated by exchange of on-shell gravitons *i.e.* those Kaluza-Klein levels satisfying $q^2 = \vec{n} \cdot \vec{n}/R^2$. On the other hand, its real part follows from exchange of off-shell gravitons. For spacelike propagator, the scattering amplitude (21) is real since in this channel Kaluza-Klein levels cannot come on shell.

The first line of $\mathcal{A}(k^2)$ in (19), except for the overall $1/f_R(0)$ factor in front, is identical to virtual graviton exchange amplitude computed within the ADD setup. The stress tensors of the on-brane systems a and b contribute to the transition amplitude via their contractions $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$ and via the multiplication of their traces $T_{\mu}^{(a)\mu}T_{\nu}^{(b)\nu}$. While the former is effective for any two systems of particles [5, 6], the latter can exist only for systems possessing conformal breaking [11].

The second line of (19), induced by the exchange of a massive graviton, is completely new in that it exists neither in ADD [5, 6] nor in $f(\mathcal{R})$ gravity setups. The presence of the operator $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$ in this novel contribution proves particularly useful for distinguishing this general modification of gravity from $f(\mathcal{R})$ theory since the latter cannot induce brane-localized operators which involve contractions of the stress tensors.

The third line of (19) is generated by exchange of the scalar graviton in the system. Its contribution always involves traces of the stress tensors, and thus, for it to significantly influence a scattering process conformal invariance must be broken strongly (masses of the brane-localized fields must be a significant fraction of \overline{M}_D), as has been analyzed in detail elsewhere [11].

It may be of practical use to illustrate how novel structures induced by $f(\mathcal{R}, P, Q)$ gravity compare with the ones already present in the ADD setup. As has been emphasized above, the

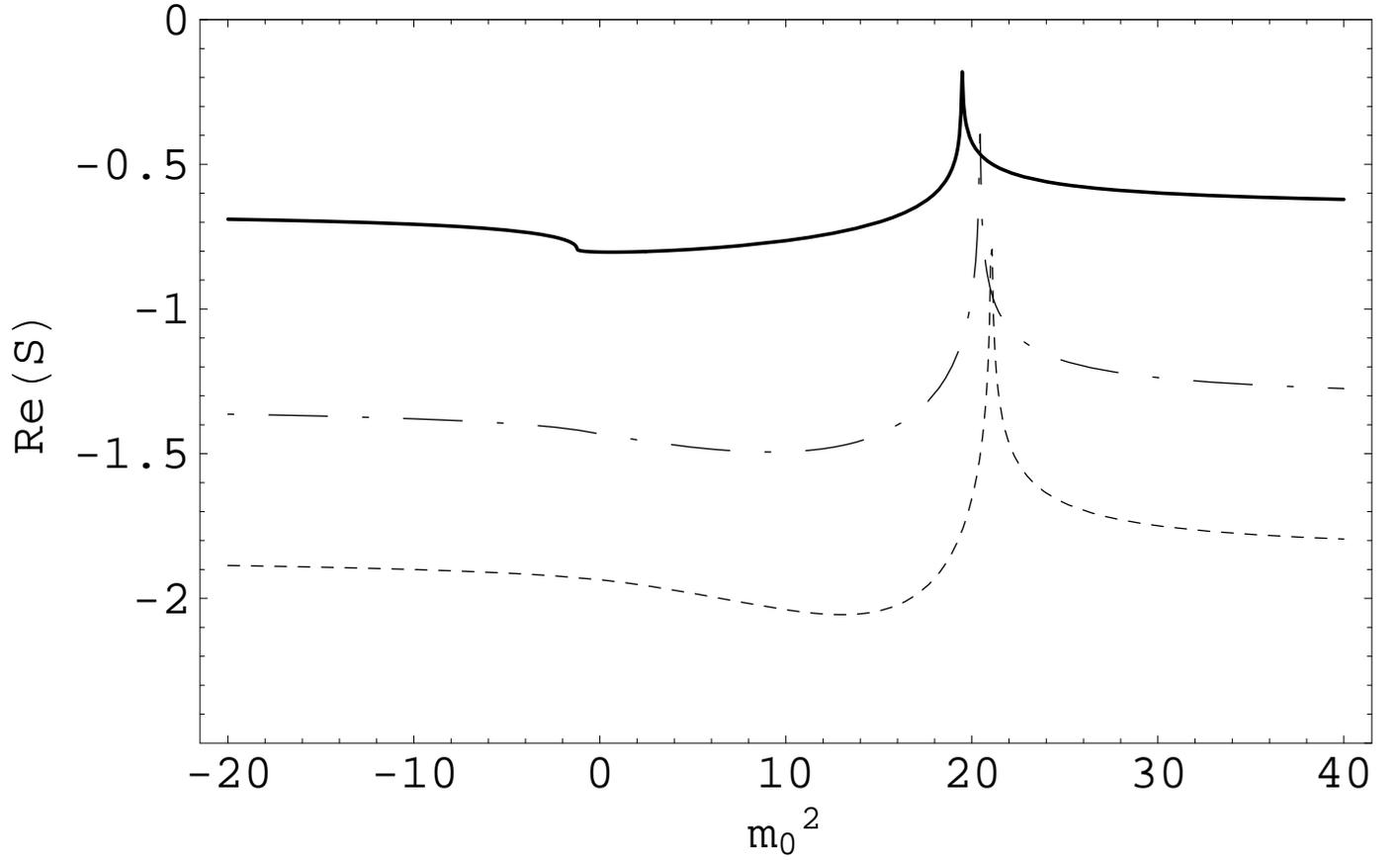


Figure 3: The dependence of $\text{Re}[S(k^2)]$ on m_0^2 for $k^2 = (1 \text{ TeV})^2$, $\Lambda = \overline{M}_D = 5 \text{ TeV}$, $m_1^2 = -10 \text{ TeV}^2$, and $\delta = 3$ (solid curve), $\delta = 5$ (dot-dashed curve) and $\delta = 7$ (short-dashed curve). In the plot m_0^2 varies from -20 TeV^2 up to 40 TeV^2 , and $\text{Re}[S(k^2)]$ flattens for large $|m_0^2|$.

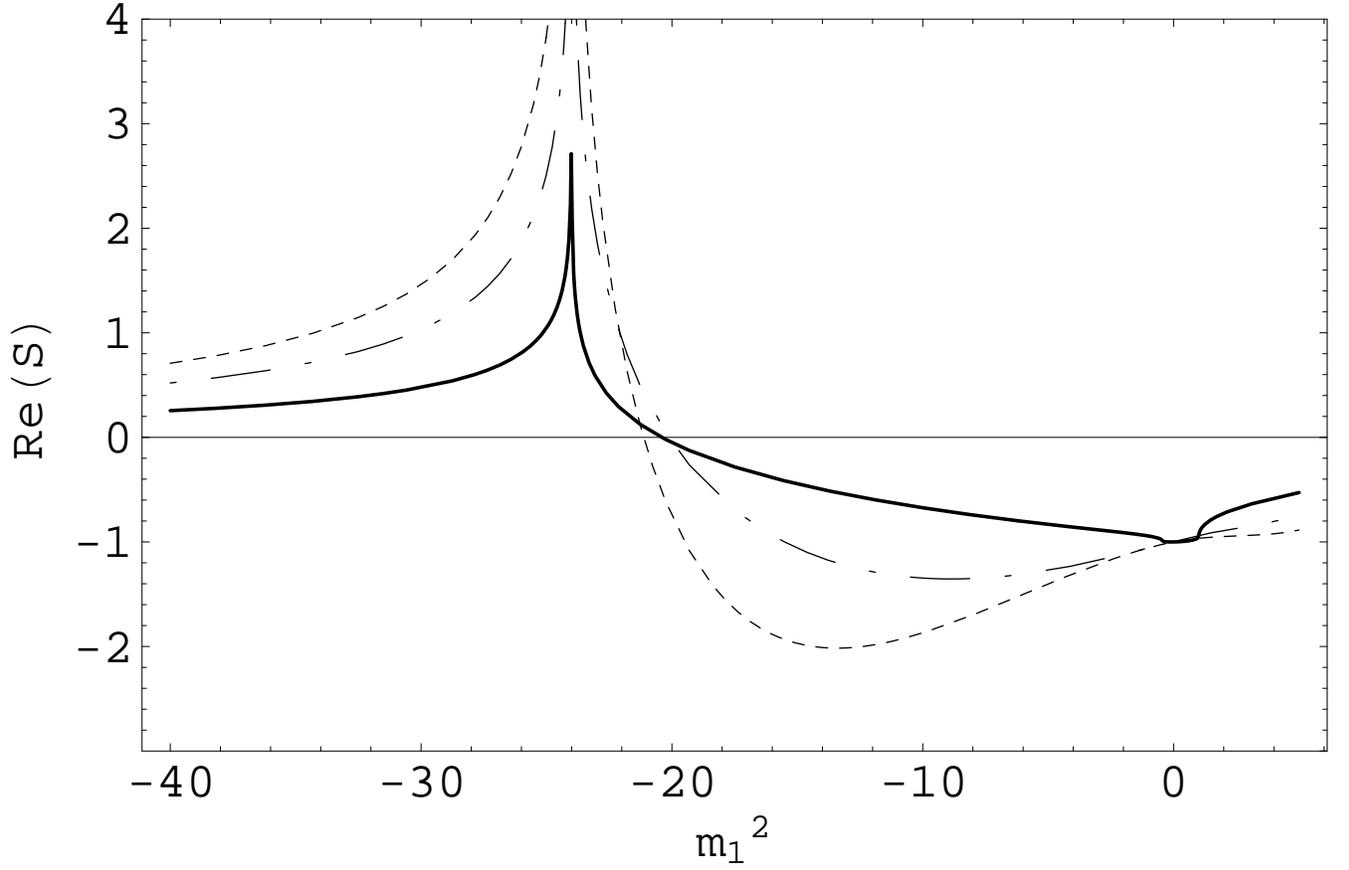


Figure 4: The dependence of $\text{Re}[S(k^2)]$ on m_1^2 for $k^2 = (1 \text{ TeV})^2$, $\Lambda = \overline{M}_D = 5 \text{ TeV}$, $m_0^2 = 5m_1^2$, and $\delta = 3$ (solid curve), $\delta = 5$ (dot-dashed curve) and $\delta = 7$ (short-dashed curve). In the plot m_1^2 varies from -40 TeV^2 up to $+5 \text{ TeV}^2$.

higher-curvature gravity theory under concern modifies the coefficients of both $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$ and $T_{\mu}^{(a)\mu}T_{\nu}^{(b)\nu}$. The former is particularly useful for collider searches as well as effective operators at low-energies since it does not require systems a and b to consist of massive brane matter. In this respect, it could be useful to dwell on the coefficient of $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$ for determining how $f(\mathcal{R}, P, Q)$ gravity contribution compares with the ADD prediction. This we do by plotting the real and imaginary parts of

$$R(k^2) = -\frac{\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_1^2}}\right)}{\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2}}\right)} \quad (22)$$

as a function of m_1^2 by taking, in accord with the future collider searches, $k^2 = (1 \text{ TeV})^2$ and $\Lambda = \overline{M}_D = 5 \text{ TeV}$. Their variations are plotted in Figs. 1 and 2 where m_1^2 is let vary from -30 TeV^2 up to $+10 \text{ TeV}^2$ for each number of extra dimensions considered: $\delta = 3$ (solid), $\delta = 5$ (dot-dashed) and $\delta = 7$ (short-dashed). These figures make it clear that massive graviton (a ghostly tensor mode special to $f(\mathcal{R}, P, Q)$ gravity) exchange significantly dominates, if not competes, the massless graviton (the only propagating mode in ADD setup) exchange when the former is a tachyon with mass-squared $\sim -0.5\Lambda^2$ (excluding the rather narrow peak at $m_1^2 = -24 \text{ TeV}^2$ which corresponds to resonating of the transition amplitude by Kaluza-Klein levels with mass-squared $= k^2 - m_1^2 = \Lambda^2$). The ghostly nature of the massive graviton affects only the sign of (22) whereas its tachyonic nature gives rise to a spectacular enhancement in $R(k^2)$ which in turn enables one to disentangle $f(\mathcal{R}, P, Q)$ gravity effects from those of the Einstein gravity in high-energy collider environment. From (12) it is clear that a negative m_1^2 implies a positive $f_P(0) + 4f_Q(0)$ since $f_R(0)$ must be positive for preventing massless graviton from becoming a ghost.

We now turn to discussion of the coefficient of $T_{\mu}^{(a)\mu}T_{\nu}^{(b)\nu}$ in (19) for determining impact of $f(\mathcal{R}, P, Q)$ gravity relative to Einstein gravity. We quantify analysis by examining the ratio

$$S(k^2) = \frac{\text{Coefficient of } T_{\mu}^{(a)\mu}T_{\nu}^{(b)\nu} \text{ from 2nd and 3rd lines of (19)}}{\text{Coefficient of } T_{\mu}^{(a)\mu}T_{\nu}^{(b)\nu} \text{ from 1st line of (19)}} \quad (23)$$

in a way similar to (22). This quantity does not have a direct meaning in interpreting the scattering rates of massive brane matter as they receive contributions from $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$,

too. Nevertheless, for determining effects of higher-curvature gravity it could be instructive to determine how $S(k^2)$ depends on various model parameters. Depicted in Fig. 3 is the variation of $\text{Re}[S(k^2)]$ with m_0^2 for $m_1^2 = -10\text{TeV}^2$, $k^2 = (1\text{TeV})^2$ and $\Lambda = \overline{M}_D = 5\text{TeV}$ for each number of extra dimensions considered: $\delta = 3$ (solid), $\delta = 5$ (dot-dashed) and $\delta = 7$ (short-dashed). The figure shows it manifestly that $f(\mathcal{R}, P, Q)$ gravity contributions completely dominate the one found in the ADD setup for large negative m_0^2 . The extra gravi-scalar, not found in Einstein gravity, results in an enhancement in scattering amplitudes of massive brane matter.

We also plot m_1^2 dependence of $S(k^2)$ in Fig. 4 by taking $m_0^2 = 5m_1^2$ and keeping other parameters as in Fig. 3. Here, unlike the case study depicted in Fig. 3, gravi-particles decouple from the spectrum at large m_1^2 due to the fact that m_0^2 varies in proportion with m_1^2 . The figure suggests that $f(\mathcal{R}, P, Q)$ gravity contribution is particularly enhanced in negative m_1^2 domain especially when $m_1^2 \sim -25\text{TeV}^2$. From the figures and their accompanying discussions one therefore concludes that, higher-curvature gravity theory (4) provides additional gravi-particles and they result in significant enhancements in virtual gravi-particle exchange amplitudes with respect to both Einstein [5, 6] and $f(\mathcal{R})$ gravity theories.

In the next section we will survey and briefly discuss certain observables (concerning especially collider searches for extra dimensions) in light of the virtual gravi-particle exchange amplitude (19) and its discussions and illustrations via the figures.

3 Effects of Gravi-Particle Exchange on Brane Processes

It might be instructive to discuss in some length certain higher dimensional operator structures which can leave significant impact on scatterings among brane-localized matter. From (19) it is clear that virtual gravi-particle exchange between two systems of brane matter leads to dimension-8 operators $T_{\mu\nu}^{(a)}T^{(b)\mu\nu}$ and $T_\mu^{(a)\mu}T_\nu^{(b)\nu}$. These additional interactions can open up novel scattering channels not found in the SM or modify the existing ones in an observable way. Therefore, they are of potential importance for collider as well as precision physics of modified gravitational interactions in higher dimensions.

It may be convenient to group and analyze effects of higher dimensional operators accord-

ing to the virtualities of the gravi-particles involved, as we do in the following subsections.

3.1 Tree-level effects of Virtual Gravi-Particles:

The tree-level gravi-particle exchange, as has been detailed in the last section, gives rise to anomalous interactions among brane matter species [5, 6, 11]. The on-brane processes are entirely tree-level ones in such processes; however, they exhibit appreciable sensitivity to gravi-particle exchange due to rather high virtualities that gravi-particles obtain via their propagation through the extra dimensions. In general, tree-level gravi-particle exchange induces various modifications in scattering processes, and they may be detected at colliders or in other experiments [13]. At an e^+e^- collider, for instance, pair-productions of gauge bosons (*e.g.* $e^+e^- \rightarrow VV$ where $V = \gamma, Z, W$) and of fermions (*e.g.* $e^+e^- \rightarrow t\bar{t}$ or any other quark or lepton) prove particularly useful for disentangling gravi-particle effects. In fact, existing results from LEP experiments already provide precise bounds on such effects from pair-productions of gauge bosons and fermions [14]. There also exist promising scattering processes at hadron (pp collisions at the LHC and $p\bar{p}$ collisions at Tevatron [15]) and lepton-hadron (ep collisions at HERA [16]) colliders which can probe gravi-particle effects at different energy scales with different particle species. In addition, there exist various phenomena, ranging from rare decays to supernovae and to ultra high energy cosmic rays, by which one can put bounds of varying strength on extra dimensions and nature of the gravitational theory in higher dimensional bulk.

It may be useful to examine some generic scattering processes for determining their power of disentangling the $f(\mathcal{R}, P, Q)$ gravity effects. For instance, generic scattering amplitude $\mathcal{A}(\psi_a(k_1)\bar{\psi}_a(k_2) \rightarrow \psi_b(q_1)\bar{\psi}_b(q_2))$ for two identical fermions can be directly obtained via the replacements

$$\begin{aligned}
T_{\mu\nu}^{(a)}T^{(b)\mu\nu} &\rightarrow \frac{1}{8} \left[(k_1 + k_2) \cdot (q_1 + q_2) \bar{\psi}_a(k_2) \gamma^\mu \psi_a(k_1) \bar{\psi}_b(q_2) \gamma_\mu \psi_b(q_1) \right. \\
&\quad \left. + \bar{\psi}_a(k_2) (\not{q}_1 + \not{q}_2) \psi_a(k_1) \bar{\psi}_b(q_2) (\not{k}_1 + \not{k}_2) \psi_b(q_1) \right] \\
T_{\mu}^{(a)\mu} T_{\nu}^{(b)\nu} &\rightarrow m_{\psi_a} m_{\psi_b} \bar{\psi}_a(k_2) \psi_a(k_1) \bar{\psi}_b(q_2) \psi_b(q_1)
\end{aligned} \tag{24}$$

in the tree-level gravi-particle exchange amplitude (19). These replacements correspond to

s -channel gravi-particle exchange with $k = k_1 - k_2 = q_2 - q_1$, and depending on the quantum numbers of ψ_a and ψ_b it could be necessary to include t and u channel contributions, too. Its comparison with the corresponding amplitude in ADD setup [5, 6] reveals that fermion-fermion scattering now proceeds with additional structures provided by second and third lines of (19). They can compete in size with the ADD amplitude for certain parameter values, especially when the scattering energy $\sqrt{k^2}$ compares with the new gravitational scales m_0 or m_1 [11]. A highly interesting aspect of (19) with the replacements (24) is that effects of the modified gravity (due to the massive ghostly $J = 2$ graviton) survive even in the limit of massless fermions. This is, as one recalls from [11], not the case for $f(\mathcal{R})$ gravity to which only scatterings of the massive brane matter exhibit sensitivity. This property of $f(\mathcal{R}, P, Q)$ gravity is important in that its effects can be directly probed at high-energy colliders (where colliding beams of matter are essentially massless) and effective higher-dimensional operators consisting of light fermions. Indeed, LEP-favored modes $e^+e^- \rightarrow f\bar{f}$ ($f = e, \mu, b, t, \dots$) or Drell-Yan annihilation of quarks at hadron colliders are golden modes for detecting modified gravity effects thanks to the fact that fermions (of systems a or b) under concern are massless. In general, independent of if the brane matter is massive or massless, there are certain parameter values for which $f(\mathcal{R}, P, Q)$ gravity contributions get significantly enhanced and thus become more easily observable with respect to Einstein gravity effects as can be seen from Figs. 1, 2, 3 and 4.

The $2 \rightarrow 2$ fermion scattering example above can be generalized to any other SM particle, and the relative enhancements/suppressions in their rates are always governed by (19) with supplementary illustrations given in the figures. A dedicated search for extra dimensions via virtual gravi-particle exchange processes requires a global analysis of various collider processes [14, 15, 16]. The most advantageous aspect of $f(\mathcal{R}, P, Q)$ gravity is the separability of massive ghostly graviton contribution from those of the remaining gravi-particles via the measurement of the scattering rates of massless (or more precisely much lighter than the fundamental scale of gravity) matter species.

3.2 Loop-level effects of Virtual Gravi-Particles:

The loop-level processes on the brane proceed with looping brane matter and/or gravi-particles where the latter are now virtual both in ordinary and extra dimensions. These effects can give rise to corrections to the existing SM amplitudes as in, for instance, electroweak precision observables (particle self energies, interaction vertices, box diagrams) and rare decays [5, 6, 11]. In fact, the dedicated analysis of [17] shows that gravi-particle loop effects can become more important than their tree-level effects since they can induce potentially important dimension-6 operators with double gravi-particle exchange. This lower-dimension operator can arise in fermion, gauge boson as well as Higgs sectors of the SM.

For illustrating the impact of $f(\mathcal{R}, P, Q)$ gravity, consider dimension-6 four-fermion operator $(1/2)\bar{f}\gamma_\mu\gamma_5 f \bar{f}'\gamma^\mu\gamma_5 f'$ (f, f' standing for light quarks or leptons) which has been shown to follow from double gravi-particle exchange in [17]. A direct calculation shows it is quite sensitive to exchange novel propagating degrees of freedom in higher curvature gravity. Indeed, for massless fermions, for instance, coefficient of this operator for $f_R(0) = 1$ is 1.9, 2.6, and 2.7 times larger than the coefficient of the same operator computed in the ADD setup [17] for $m_1^2 = -14 \text{ TeV}^2$, $\Lambda = 5 \text{ TeV}$ and $\delta = 3, 5$, and 7 extra dimensions. The reason for this, as illustrated in Figs. 1 and 2, is the enhancement of $f(\mathcal{R}, P, Q)$ gravity contributions compared to Einstein gravity at this specific value of m_1^2 . Clearly, existing experimental results on contact interactions, dijet and dilepton production processes as well as lepton-hadron scattering rates can put stringent limits on the model parameters $\delta, \Lambda, \overline{M}_D, m_1^2$ and $f_R(0)$. The analysis of [17] shows that the strongest bounds come from LEP results on contact interactions [18].

Repeating, at the loop-level gravi-particle exchange gives rise to observable modifications on various phenomena testable at the present and future collider studies. Therefore, essentially what remains to be done is to perform a global analysis of the observables so as to achieve bounds or exclusion limits on $f(\mathcal{R}, P, Q)$ gravity parameters.

3.3 Effects of Real Gravi-Particles:

In addition to their virtual effects just mentioned, the gravi-particles can decay into brane-matter or can be produced by scatterings among the brane matter [5, 6, 11]. While the

former plays a crucial role in cosmological and astrophysical contexts, the latter constitutes one of the most important signatures of extra dimensions at colliders in that gravi-particle emission from the brane gives rise to scattering processes with single missing energy signal, and thus, it is of fundamental importance for distinguishing supersymmetric models from the extra dimensional ones. In these real gravi-particle involving processes the presence of bulk masses for tensor and scalar modes modify decay signatures significantly, as has been detailed in [11].

For a clearer view of the effects of gravi-particle decay/emission it proves useful to refer to their loop effects. Indeed, the Z boson self-energy, for example, represents, via the optical theorem, the Drell-Yan production of gravi-particles and Z boson at lepton (via $e^+e^- \rightarrow Z^* \rightarrow \text{gravi-particle} + Z$ annihilation) or hadron (via $q\bar{q} \rightarrow Z^* \rightarrow \text{gravi-particle} + Z$ annihilation) colliders. The main novelty brought about by $f(\mathcal{R}, P, Q)$ gravity is the production of $J = 2$ ghost and the scalar mode when the center-of-mass energy of the collider is sufficiently large. This phenomenon reflects by itself a sudden change in the number of events (similar to opening of W^+W^- channel at LEP experiments). The dominant contribution to gravi-particle emission comes from Kaluza-Klein levels in the vicinity of $R^2(M_Z^2 - m_{\text{gravi-particle}}^2)$. However, one here notes an important aspect of gravi-particle decay/emission processes: For such processes all gravi-particles must be fields with positive semi-definite mass-squareds and hence, as in $f(\mathcal{R})$ gravity [11], one does not expect significant contributions from gravi-particles other than $J = 0$ graviton (see the figures Fig. 1-4).

4 Conclusion

In this work we have discussed phenomenological implications of $f(\mathcal{R}, P, Q)$ gravity in higher dimensional spacetimes with large extra spatial dimensions. In Sec. 2 we have expanded action around a flat background and computed the propagator. Moreover, after determining the propagating degrees of freedom and virtual gravi-particle exchange amplitude we have provided a detailed and comparative analysis of the contributions of $f(\mathcal{R}, P, Q)$ and Einstein gravity. We have therein witnessed important enhancements/suppressions, as illustrated via the figures, brought about by the higher-curvature gravity theory considered.

In Sec. 3 we have analyzed effects of virtual and real gravi-particles on the scatterings among the brane matter. This section has shown that there exist a number of laboratory and astrophysical processes a global analysis of which can provide important information about the nature of the gravitational theory in the higher dimensional bulk. The discussions therein suggest that $f(\mathcal{R}, P, Q)$ gravity theories with finite $f_{RR,P,Q}(0)$ can induce potentially important effects testable at future collider studies.

The higher-curvature gravity theory discussed in this work offer various signatures which distinguish it from the Einstein and $f(\mathcal{R}, P, Q)$ gravity theories, and a global survey of laboratory, astrophysical and cosmological observables (see [1] can reveal presence or absence of such higher-curvature generalizations of the Einstein-Hilbert action. Indeed, affirmative or negative, the answer will be crucial for establishing the gravitational interactions beyond Fermi energies and may pave the road to a full understanding of quantum gravity.

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