ENTROPIC TUNNELING TIME

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(D. Demir and T. Güner, Annals of Physics **386** (2017) 291)

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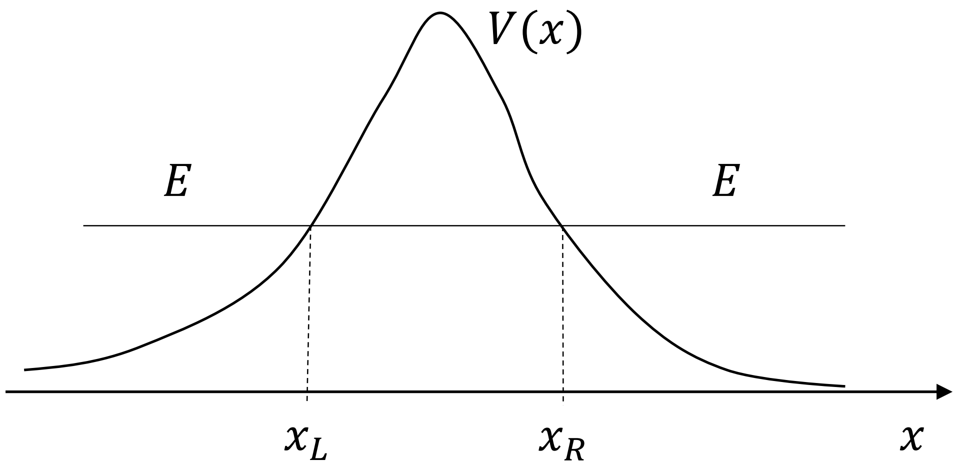
Tunneling



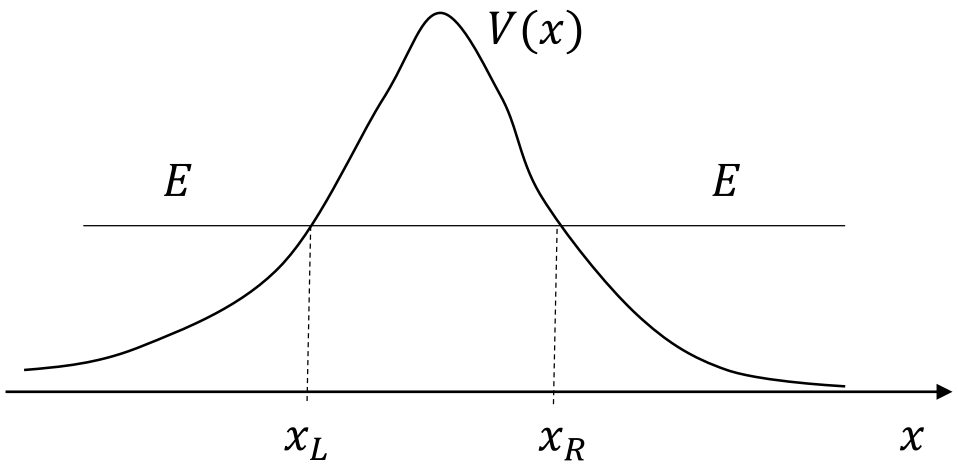
« A particle of mass m and energy E,

incident on a potential barrier $V(x)$,

disappears at x_L and reappears at x_R,

with the same energy»

«The Problem»



How long does it take to get from x_L to x_R ?

Experiment: measured to be nonzero (*e.g.* He ionization)

Theory: various contradictory formulae

Classical Dynamics

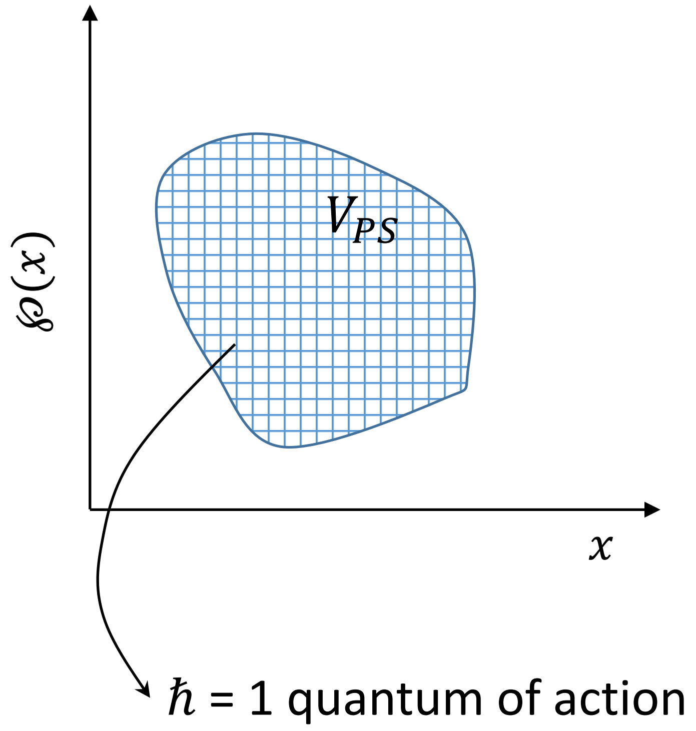
Momentum is pure imaginary: $\sqrt{2m\,(E - V(x)\,)} = -i\,\sqrt{2m\,(V(x) - E)} \equiv -i\,\wp(x)$

Classical time is pure imaginary: $(\Delta t)_{cl} = \int_{x_L}^{x_R} \frac{m\,dx}{-i\,\wp(x)} = i\tau_c$

$\therefore$ Imaginary time $\sim$ Inverse temperature $\Longrightarrow$ Under-barrier dynamics might be statistical

Microstates

Phase space volume: $V_{\mathrm{PS}} = \int_{x_L}^{x_R} \wp(x)\, dx$



Bohr-Sommerfeld Quantization: $\frac{2\,V_{PS}}{\hbar}$ = an integer

2×(# of quantum actions) $= 2\,\Phi$ = # of microstates

Entropy

$S = k_B \langle$ log (# of microstates) $\rangle = k_B \langle \log (1 + 2\,\Phi) \rangle$

which equals

$S = k_B\, p \log (1 - \log p)$

with

$p = e^{-2\Phi}$ = probability that particle starts at x_L and arrives at x_R directly

Temperature

$$\frac{1}{T} = \frac{\partial S}{\partial E} = - \frac{2k_B\tau_c}{\hbar} e^{-2\,\Phi} \left(\frac{1}{1+2\,\Phi} + \log\frac{1}{1+2\Phi}\right)$$

thermal energy $k_B T$ varies with particle properties (m, x_L, x_R, E)

Entropic Tunneling Time

$$\Delta t \equiv \frac{\hbar}{2 \times \mathcal{T} \times (2\,\pi\, k_B\, T)}$$

transmission probability ($\mathcal{T} = \frac{1}{\mathrm{Cosh}^2\, \Phi}$ in WKB)

$$\frac{d^2\psi}{dx^2} = \left(\frac{\wp(x)}{\hbar}\right)^2 \psi$$

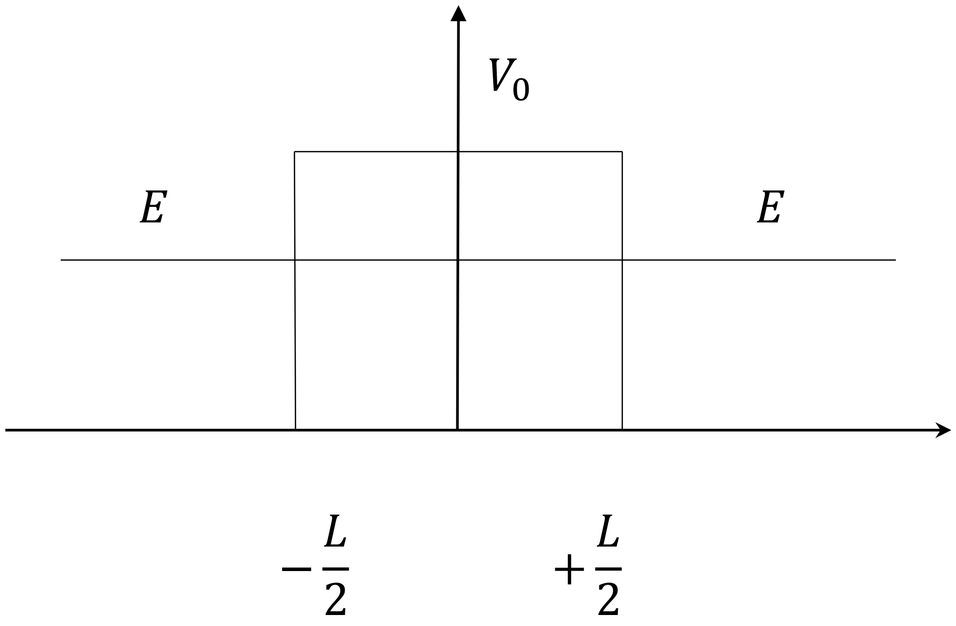
ETT vs. Known Tunneling Time Formulae

Wide barrier limit ($L \to \infty$):

$$(\Delta t)_{ETT} \to \infty$$

Phase time $(\Delta t)_{\varphi} \to \frac{\hbar}{E}\sqrt{\frac{E}{V_0 - E}}$

Dwell time $(\Delta t)_D \to \frac{\hbar}{V_0}\sqrt{\frac{E}{V_0 - E}}$



Confronting ETT with Experiment

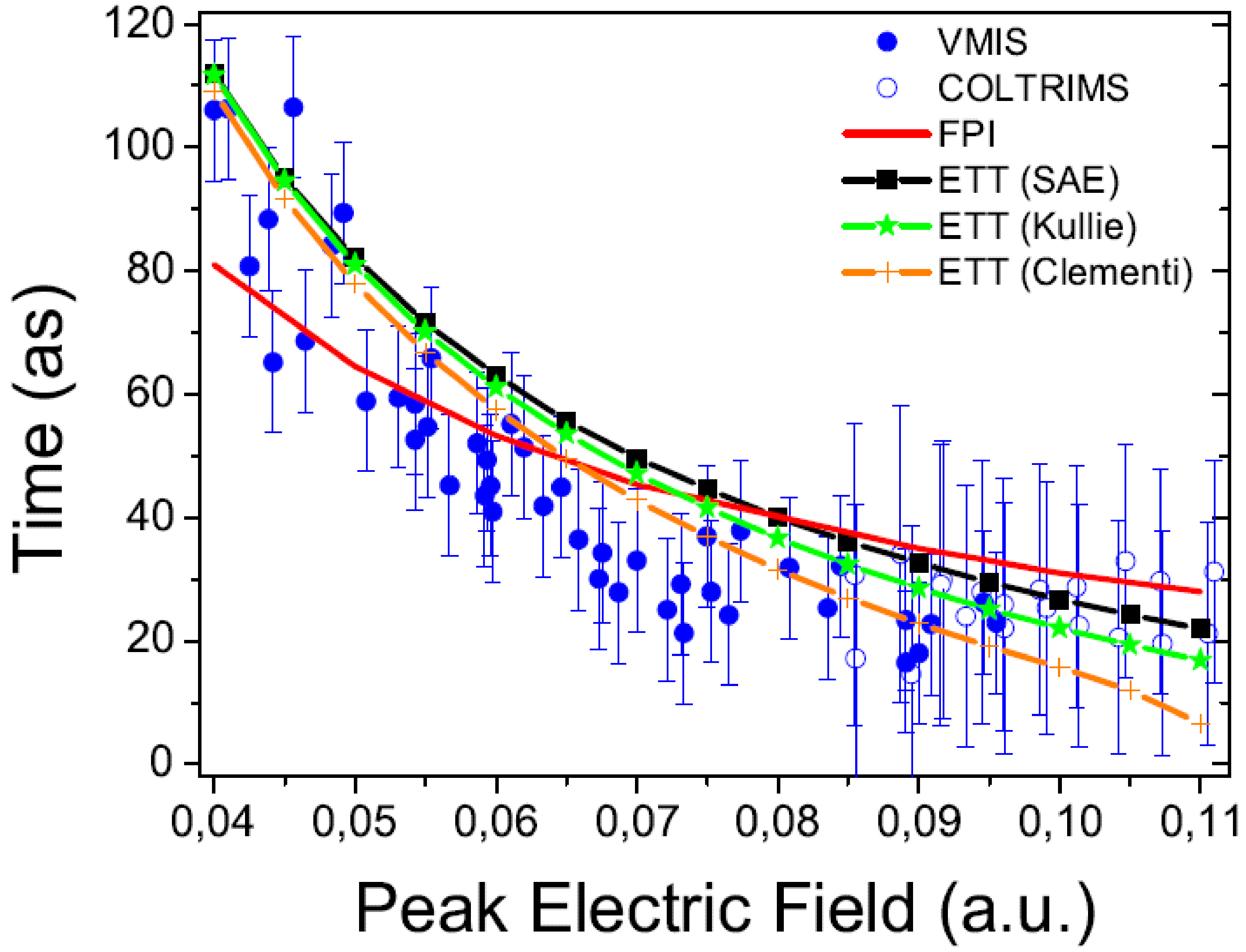
Effective potential for a single electron at a distance x from the He^{+} ion

$$V(x) = -\frac{Z_{eff}(x)}{x} - \mathcal{E}\, x \qquad (\,\mathcal{E} = \text{laser (electric) field peak value})$$

- $Z_{eff}(x) = 1 + 1.231\, e^{-0.662x} - 1.325\, e^{-1.236x} - 0.231\, e^{-0.48x}$ (SAE)
- $Z_{eff}(x) = 1.375$ (Kullie)
- $Z_{eff}(x) = 1.6875$ (Clementi)

$\mathcal{E} = \mathbf{0.04}$	x_L	x_R	τ_c (as)	ETT (as)
SAE	1.24	21.43	833.82	113.08
Kullie	1.64	20.96	850.73	111.75
Clementi	2.05	20.55	856.49	109.14

Confronting ETT with Experiment



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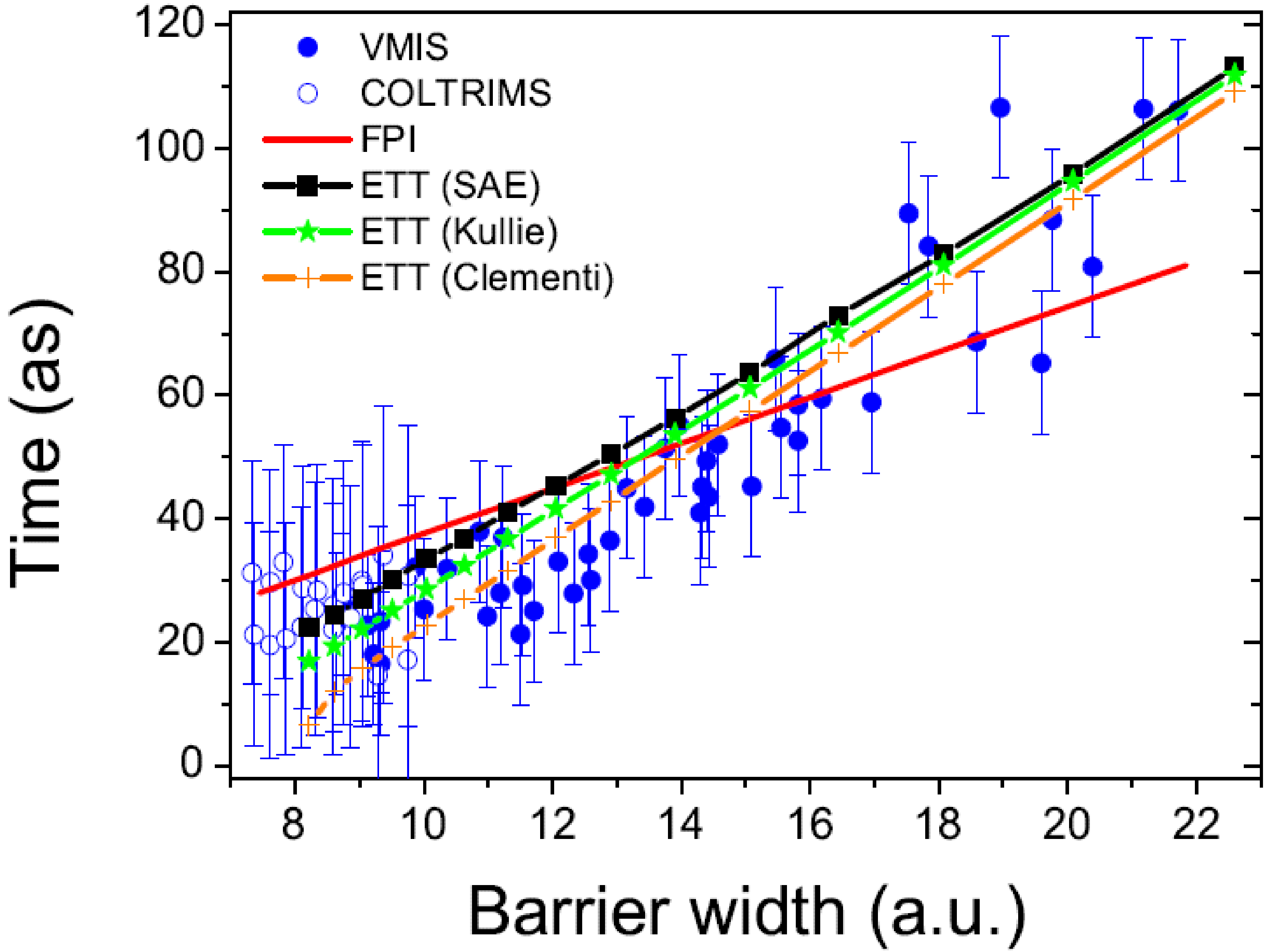
Confronting ETT with Experiment



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Future Prospects

1. ETT might be a working formulation of tunneling time.

2. ETT needs be confronted with more data (not available yet)

3. ETT, applied to photon tunneling, can be relevant in photonics.