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Radiative corrections to $\nu_\mu e$ scattering

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Received 30 May 1995, in final form 20 July 1995

Abstract. The electroweak radiative corrections to $\nu_\mu e$ scattering are calculated and the vector and axial couplings are obtained. The theoretical results are in good agreement with the experimental data.

1. Introduction

The increase in the precision of the measurements in LEP and SLC has opened a new stage of electroweak interactions. The precision of these measurements has reached a level at which the quantum effects could be measured. With this motivation the theoretical calculations of radiative corrections has become one of the important problems of particle physics.

There have been many theoretical attempts in the literature devoted to the calculation of electroweak radiative corrections to various processes (see, for example, the recent reviews [1]). Among these, the approach developed by Okun and his collaborators [2] is very suitable for comparison with the experimental data because any physical quantity is given by analytic expressions.

For the analysis of the electroweak data one starts with a set of input parameters. In [2] the input parameters are chosen as

\begin{align*}
\tilde{\alpha} &= \alpha(M_Z^2) = \frac{1}{128.78} \\
M_Z &= 91.175 \text{ GeV} \\
G_\mu &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}
\end{align*}

The phenomenological angle $\theta$ which was first introduced in [3] and widely used in [2], is defined as

$$
\tan^2 \theta = \frac{\pi \tilde{\alpha}}{\sqrt{2} G_\mu M_Z^2}.
$$

The basic reason for choosing this set of parameters is due to the fact that they are very accurately determined in the experiments.

In this paper we calculate the radiative electroweak corrections to $\nu_\mu e$ scattering in the framework of the approach developed in [2]. Note that part of the calculations have already been performed in [4]; here we extend them by including all the vertex, self-energy and box diagram contributions. In the appendix we demonstrate the cancellation of the divergences.

The radiative corrections to the $\nu_\mu e$ scattering are described by the diagrams in figure 1.

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The tree level amplitude for this scattering process is given by
\[ M_0 = -i \frac{f_0^2}{4 M_Z^2} \frac{1}{2} \bar{v}_\mu \gamma_\nu (1 - \gamma_5) v_\mu \gamma^\mu [-\frac{1}{2} + 2 s^2 + \frac{1}{2} \gamma_5] e_0 \] (5)

where the subscript 0 indicates that the quantities under consideration are bare ones.

Taking into account all the radiative corrections, we obtain the one-loop corrected $\nu_\mu e$ scattering amplitude
\[ M = -i \frac{f^2}{4 M_Z^2} q_{ve} \bar{v}_\mu \gamma_\nu (1 - \gamma_5) v_\mu \gamma^\mu [-\frac{1}{2} + 2 s^2_{ve} + \frac{1}{2} \gamma_5] e. \] (6)

Here $q_{ve}$ and $s_{ve}$ are functions of $q^2$ and are given by
\[ s_{ve}^2(q^2) = s_0^2 + sc \left\{ \Pi_Y(q^2) + 4sc \frac{M_2^2}{q^2} \Gamma_Y(q^2) + 4sc (-a_e \Gamma_Y^2(q^2) + v_e \Gamma_Y^2(q^2)) + B_s \right\} \] (7)
\[ q_{ve} = \frac{1}{2} + 1/2 (\Delta f_0 + \Delta v + \Delta M_Z) + 1/2 \Pi_Z(q^2) + 2sc (\Gamma_Y^2(q^2) - \Gamma_A^2(q^2)) + B_e. \] (8)

The definition of the quantities in equations (5)–(8) are given by the following expressions:
\[ s_0^2 = s^2 - \frac{3 \bar{s}}{16 \pi} \frac{c^2}{s^2(c^2 - s^2)} V_m(t, h) + c^2 (\Pi_Z(M_2^2) - \Pi_W(M_W^2)) \] (9)
Radiative corrections to $\nu_e e$ scattering

where $V_m(t, h)$ could be read from [2] and $t = m^2 / M_Z^2$ and $h = M^2 / M_Z^2.$

Now we write the unrenormalized self-energy $\Pi$ as follows:

$$\Pi = \text{div} \Pi + f_\Pi$$

where $\text{div} \Pi$ shows the divergent part of $\Pi$ and is given in the appendix for the cases of interest and $f_\Pi$ is the finite part of $\Pi$. In the same manner we make a separation of the vertex function $\Gamma$

$$\Gamma = \text{div} \Gamma + f_\Gamma.$$  

The terms in $\varphi_{\nu e}$ need further explanation: $\Delta f_0$ is the one-loop contribution to $f_0^2$ and reads

$$\Delta f_0 = \Pi_2(M_Z^2) - \Pi_\nu(0) - D$$

where $D$ is defined in [2] and is given by

$$D = \text{div} D + \frac{\bar{\alpha}}{4\pi s^2} \left( 6 + \frac{7 - 4s^2}{2s^2} \ln(c^2) \right)$$

where $\text{div} D$ is given in the appendix.

$\Delta_\nu$ comes from the neutrino wavefunction renormalization [5]

$$\Delta_\nu = \text{div} \Delta_\nu - \frac{\bar{\alpha}}{4\pi} \left\{ \ln \left( \frac{m_e^2}{M_Z^2} \right) - \ln(c^2) - 2 \ln \left( \frac{m_e^2}{\lambda^2} \right) + 4 - (v_e + a_e)^2 (\ln(c^2) + \frac{1}{2}) - \frac{1}{4s^2} \right\}$$

where $\lambda$ is the infrared cut-off on the photon invariant mass.

The remaining term $\Delta M_Z$ comes from $1/M^2$ and reads

$$\Delta M_Z = -\Pi_2(M_Z^2).$$

Note that the terms coming from the renormalization of the electron wavefunction are already included in the expressions for the vertices.

Let us now give the explicit expressions for the finite parts of the self-energies appearing in equations (7)-(9):

$$f_2(M_Z^2) = \frac{\bar{\alpha}s^2}{\pi c^2} \left( \frac{Q^2}{Q_1^2 - \frac{Q_1}{2s^2}} \right) (\ln(t/c^2) + (1 + 2t)F_t(t) - \frac{1}{3})$$

$$+ \frac{\bar{\alpha}s^2}{\pi c^2} \left( \frac{Q^2}{2s^2} + \frac{Q_b}{2s^2} \right) (2 \ln(b/c^2) + \frac{5}{3})$$

$$+ \frac{\bar{\alpha}}{8\pi s^2 c^2} (1 - t)F_t(t) + 3 \ln(t) - (2 - 3t) \ln(c^2) + \frac{5}{3}$$

$$+ \frac{N_c\bar{\alpha}}{3\pi s^2 c^2} (-\ln(c^2) + 5/3)(1/4 - s^2/2 + s^4(Q^2_u + Q^2_d))$$

$$+ \frac{\bar{\alpha}}{4\pi s^2} \left[ -0.58 - \frac{h}{8c^2} + \frac{3h}{4c^2(1 - h)} \ln(h) + \left( \frac{1}{c^2} - \frac{h}{3c^2} + \frac{h^2}{12c^2} \right) F_b(h) \right]$$

$$f_W(M_W^2) = \frac{\bar{\alpha}}{4\pi s^2} \left[ \left( 1 - \frac{3t}{2c^2} \right) \ln(t/c^2) - \frac{t^3}{2c^6} \ln(|t-c^2|) + 5/3 - \frac{t}{c^2} - \frac{t^2}{2c^4} \right]$$

$$+ \frac{N_c\bar{\alpha}}{12\pi s^2} + \frac{\bar{\alpha}}{4\pi s^2} \left[ -1.76 - \frac{h}{6c^2} + \frac{3h}{4c^2(1 - h)} \ln(h/c^2) \right]$$
\begin{align}
\gamma_{\nu Z}(q^2) &= \frac{\bar{\alpha}}{4\pi} \left[ \frac{-1 + 4s^2}{3sc} \sum_{\nu e, \mu} \left( 1 + \frac{2m^2_{e^4}}{q^2} F(q^2, m_t, m_t) \right) + \frac{7}{9sc} \left( 1 - \frac{8s^2}{3} \right) \right] \\
&\hspace{1cm} + \frac{\bar{\alpha}}{\pi sc} \left[ \left( \frac{Q^2}{4} - s^2 Q^2_t \right) \ln(t/c^2) + \left( \frac{Q^2}{4} + s^2 Q^2_b \right) \ln(b/c^2) \right] \\
&\hspace{1cm} + \frac{N_c}{3} \left( \frac{1}{c^2} - s^2 (Q^2_b + Q^2_b) \right) \ln(c^2) \\
\gamma_{W}(0) &= \frac{\bar{\alpha}}{4\pi} \left[ -\frac{3t}{4s^2c^2} + \frac{3}{4s^2(1 - c^2/h)} \ln(c^2/h) - h/8c^2 + 0.85 \right] \\
\gamma_{Z}(q^2) &= \frac{\bar{\alpha}}{4\pi} \left[ -2h \ln(h/c^2) + 2 \ln(c^2) + (10 - 2h) \left( 1 - \frac{h + 1}{h - 1} \right) \ln(h) - \ln(h/c^2) \right].
\end{align}

Here \( b = m_t^2/M_Z^2 \) and the analytic expressions for functions \( F_{\nu e}(t), F_{\nu e}(h) \) could be found in \cite{2} and \( F(q^2, m_1, m_2) \) in \cite{5}.

In \( \nu_{\mu} e \) scattering \( q^2 \) is very small as compared to \( M_Z^2 \), so we neglect terms of the order \( q^2/M_Z^2 \) and \( m_t^2/M_Z^2 \) unless a divergence problem occurs. However, in our case \( q^2 \) and the light fermion masses are comparable, so we take into account the terms \( m_t^2/q^2 \).

After completing the listing of the finite parts of the self-energies we now turn to the finite parts of vertices \cite{5}:

\begin{align}
\gamma_{\nu Z}(q^2) &= F_{\nu}(q^2) \\
\gamma_{W}(q^2) &= -\frac{\bar{\alpha}}{4\pi s^2c} \left( \frac{c(1 - s)}{8s^2} \right) + F_{W}(q^2) \\
\gamma_{A}(q^2) &= -\frac{\bar{\alpha}}{4\pi s^2c} \left( \frac{c(1 - s)}{8s^2} \right) + F_{A}(q^2) \\
\gamma_{V}(q^2) &= \frac{\bar{\alpha}}{4\pi sc} \left\{ -\frac{1}{8s^2c^2} - \frac{1}{4s^2c^2} 2 \ln(c^2) - \frac{1}{4s^2} \right. \\
&\hspace{1cm} \left. + \ln(M_Z^2/m_e^2) + \frac{3}{2} - 2 \ln(m_e^2/\lambda^2) \right\} + F_{V}(q^2).
\end{align}

The symbols in these expressions read

\begin{align}
F_{\nu}(q^2) &= -\frac{\bar{\alpha}}{4\pi s^2c} \nu_{\nu} c s A_3 \\
F_{W}(q^2) &= \frac{\bar{\alpha}}{4\pi} \nu_{\nu} \left[ A_1 + \frac{1}{s^2} (A_2/2 - c^2 A_3) \right] \\
F_{A}(q^2) &= \frac{\bar{\alpha}}{4\pi} \left\{ \nu_{\nu} \left( \nu_{\nu}^2 + 3a_e^2 \right) A_4 \right. + \left. \frac{1}{s^2} \left[ \frac{\nu_{\nu} + a_e}{4} A_5 - a_e c^2 A_6 \right] \right\} \\
F_{V}(q^2) &= \frac{\bar{\alpha}}{4\pi} \left\{ a_e \left( a_e^2 + 3 \nu_{\nu}^2 \right) A_4 \right. + \left. \frac{1}{s^2} \left[ \frac{\nu_{\nu} + a_e}{4} A_5 - a_e c^2 A_6 \right] \right\}.
\end{align}

Here and in the previous formulae the vector and axial vector couplings are defined as follows

\begin{align}
\nu_{\nu} &= \frac{1}{4sc} \hspace{1cm} a_e = -\frac{1}{4sc} \hspace{1cm} \nu_{\nu} = \frac{-1 + 4s^2}{4sc}.
\end{align}
The functions $A_i, i = 1, \ldots, 6$ (see also [6]) are given by

$$A_1 = -\frac{q^2}{M_W^2} \left( \frac{2}{3} \ln \left( -\frac{q^2}{M_W^2} \right) - \frac{11}{9} \right)$$

$$A_2 = -\frac{q^2}{M_W^2} \left( \frac{2}{3} \ln \left( -\frac{q^2}{M_W^2} \right) - \frac{11}{9} - \frac{8 m^2_\mu}{3 q^2} + \frac{2}{3} \left( 1 + \frac{m^2_\mu}{q^2} \right) \beta_\mu L_\mu \right)$$

$$A_3 = A_2 + \frac{8 q^2}{9 M_W^2}$$

$$A_4 = A_2(M_W \to M_Z; m_\mu \to m_e)$$

$$A_5 = A_1(M_Z \to M_W)$$

$$A_6 = -\frac{q^2}{M_W^2} \left( \frac{2}{3} \ln \left( -\frac{q^2}{M_W^2} \right) - \frac{19}{9} \right)$$

where

$$\beta_i = \sqrt{1 - \frac{4 m^2_e}{q^2}}$$

$$L_i = \frac{1}{2} \ln \left\{ \frac{1 + \beta_i - 2 m^2_f / q^2}{1 - \beta_i - 2 m^2_f / q^2} \right\}$$

In the expressions for $s^2_{ve}(q^2)$ and $\varphi_{ve}(q^2)$ the contribution of the box diagrams (see figure 1) are denoted by $B_s$ and $B_\varphi$ respectively:

$$B_s = -\frac{1}{c_s} \left( \tilde{\alpha} \left( \frac{\tilde{\alpha}}{2 \pi s^2} + \frac{3 \tilde{\alpha}}{4 \pi s^2} v_a a_\varphi \right) \right)$$

$$B_\varphi = \frac{1}{2} \left( \tilde{\alpha} \left( \frac{\tilde{\alpha}}{2 \pi s^2} + \frac{3 \tilde{\alpha}}{8 \pi s^2} (v_a a_e ^2) \right) \right)$$

In the appendix it is shown explicitly that $s^2_{ve}$ and $\varphi_{ve}$ are finite. Now using the expressions for the self-energies and vertices we obtain

$$s^2_{ve}(q^2) = s^2 - \frac{3 \tilde{\alpha}}{8 \pi (c^2 - s^2)} V_R(q^2)$$

and

$$\varphi_{ve}(q^2) = \frac{1}{2} - \frac{3 \tilde{\alpha}}{64 \pi c^2 s^2} V_\Lambda(q^2)$$

where

$$V_R(q^2) = \frac{c^2}{2 s^2} V_m(t, h) - \frac{2(c^2 - s^2)}{3} K(q^2)$$

$$V_\Lambda(q^2) = \frac{16 \pi s^2 c^2}{3} \left\{ \frac{1}{2}(t_1 + t_2) + 1/2(\tilde{f}_Z(q^2) - \tilde{f}_W(q^2)) \right\}$$

$$+ 2sc(\tilde{f}_A^2(q^2) - \tilde{f}_W^2(q^2)) + \frac{c^2}{4 s^2}(1 - s)$$

In these expressions the bar on any quantity indicates that $\tilde{\alpha}/4\pi$ is factored out from the corresponding unbarred quantity. In (43) $t_1$ comes from the combination $\Delta f_0 + \Delta M_Z + \Pi_Z(q^2)$ and reads

$$t_1 = -\frac{1}{s^2} \left( 6 + \frac{7 - 4s^2}{2s^2} \ln(c^2) \right) + \frac{t}{12c^2 s^2} \ln(b)$$
and $t_2$ originates from the combination $2sc\Gamma_\nu^2 + \Delta_\nu$ and is given by

$$t_2 = 1/2 \left\{ 2\ln(m_e^2/M_Z^2) + \frac{3 - 2c^2}{4s^2} + \left( 1 + \frac{-1 + 2s^2}{8s^2c^2} \right) \ln(c^2) \right\}. \quad (45)$$

The term $(c^2/4s^2)(1-s)$ in $V_A$ follows from $\Gamma_A^2$ and $\Gamma_V^2$. Finally the function $K(q^2)$ in $V_R$ is given by

$$K(q^2) = c^2(f_Z(M_Z^2) - F_W(M_W^2)) + sc\left( \frac{4s^2cM_Z^2}{q^2} f^Y_V(q^2) - f^Z_V(q^2) \right) + 4sc(-aeF^Z_V(q^2) + veF^Y_V(q^2)) - \frac{1}{s^2} (2 + 3(v_c^2 + a_e^2)). \quad (46)$$

Let us note the similarity between $V_A(q^2)$ and $V_R(q^2)$ and the $V_A(t, h)$ and $V_R(t, h)$ in [2]. Although the expressions for $g_V$ and $g_A$ are similar in form in this work and in [2] the expressions for the $V$-functions are completely different.

The calculations show that $Q_{ve}(q^2)$ is practically independent of $q^2$, but $s_{ve}^2(q^2)$ has considerable dependence on $q^2$. Therefore, we calculate the average value of $s_{ve}^2(q^2)$ at fixed values of $m_t$ and $M_H$ where averaging is defined in the following manner:

$$\langle s_{ve}^2 \rangle = \frac{1}{q_{max}^2 - q_{min}^2} \int_{q_{min}^2}^{q_{max}^2} s_{ve}^2(q^2) dq^2$$

where $q_{min}^2 = 0$ and $q_{max}^2 = -2m_eE_e$ and $E_e = 25.7$ GeV in accordance with the CHARM II experiment [7].

The calculations show that the $M_H$-dependence of $\langle s_{ve}^2 \rangle$ and $\langle Q_{ve} \rangle$ is weak and they change a few per cent as $M_H$ ranges from 100 to 1000 GeV. At $m_t = 176$ GeV and $M_H = 300$ GeV we get

$$\langle s_{ve}^2 \rangle = 0.232 \pm 0.007 \quad (47)$$
$$\langle Q_{ve} \rangle = 0.4945 \pm 0.001. \quad (48)$$

According to the usual electroweak radiative correction procedure, we present the $M_H$ dependence of $1 - 4\langle s_{ve}^2 \rangle = g_V^{ve}/g_A^{ve}$ (see below) for the allowed range of top quark mass which follows from the data in the CDF and D0 collaborations [9] (figure 2). As we noted before, indeed the $M_H$ dependence of $\langle s_{ve}^2 \rangle$ is rather weak and the results are practically insensitive to $M_H$.

Now we consider the process in terms of another parametrization such that

$$M = -i G_F \sqrt{2} \bar{\nu}_\nu \gamma_5 (1 - \gamma_5) \nu_\nu \bar{\nu}_e \gamma^\alpha (g_V^{ve} - g_A^{ve}) \gamma_5 e \quad (49)$$

which was used in [8] for the interpretation of the CHARM II data.

In terms of the previous quantities the new parameters are given by

$$g_V^{ve}(q^2) = \rho(q^2)(-1 + 4s_{ve}^2(q^2)) \quad (50)$$
$$g_A^{ve}(q^2) = -\rho(q^2). \quad (51)$$

Therefore, at $m_t = 176$ GeV and $M_H = 300$ GeV we have

$$\langle g_A^{ve} \rangle = - (0.4945 \pm 0.01). \quad (52)$$

From the defining equations (50) and (51) it follows that

$$\frac{g_V^{ve}(q^2)}{g_A^{ve}(q^2)} = 1 - 4s_{ve}^2(q^2). \quad (53)$$
Using the experimental data for $g_V^{\nu e}$ and $g_A^{\nu e}$ [7]

$$g_V^{\nu e} = -0.503 \pm 0.018$$

$$g_A^{\nu e} = -0.025 \pm 0.019$$

we obtain, at $m_t = 176$ GeV and $M_H = 300$ GeV,

$$s_{\nu e}^2 = 0.2375 \pm 0.0095$$

which is in good agreement with the theoretical calculations.

Finally, we would like to make the following remark. As pointed out in [2, 10, 11] the logarithmic vertex contributions are practically cancelled by the logarithmic contributions from the $Z \rightarrow \nu$ transition but this happens only at the maximal values of $q^2$.

In conclusion, we have calculated all the radiative corrections to the $\nu_\mu e$ scattering and found the values of the vector and axial coupling constants. The accuracy of $g_A$ is high enough but the accuracy of $g_V$ is very low compared to LEP results [12].

Acknowledgment

We would like to sincerely thank Professor L B Okun for bringing our attention to this problem.

Appendix

Let us first give the divergent parts of the self-energies and the vertices defined in (10) and (11) respectively.

$$\text{div } \Pi_2(M_Z^2) = \Delta_W \left\{ \frac{\tilde{\alpha} s^2}{\pi c^2} (Q_t^2 + Q_b^2) \right. - \frac{\tilde{\alpha}}{8\pi s^2 c^2} (2 - 3t)$$

$$+ \frac{N_c \tilde{\alpha}}{3\pi s^2 c^2} \left( \frac{1}{4} - s^2/2 + s^4 (Q_u^2 + Q_d^2) \right) + \frac{\tilde{\alpha}}{4\pi s^2} (7s^2 - \frac{25}{6} + 7s^2/6c^2) \right\}$$

(57)
\[
\text{div } \nabla \omega(M_{\omega}^2) = \Delta \omega \left\{ \frac{\tilde{\alpha}}{8 \pi \xi s^2} \left(2 c^2 - 3 \xi \right) + \frac{N_c \tilde{\alpha}}{12 \pi s^2} - \frac{\tilde{\alpha}}{4 \pi s^2} \left(\frac{3}{2} \xi - s^2/c^2 \right) \right\} 
\]

\( (58) \)

\[
\text{div } \nabla \omega(q^2) = \Delta \omega \left\{ \frac{\tilde{\alpha}}{4 \pi} \frac{2 M_{\omega}^2}{q \xi} + \frac{\tilde{\alpha}}{\pi s c} \left(1/4 (Q_1 - Q_2) - c^2 (Q_1^2 + Q_2^2)\right) - \frac{N_c \tilde{\alpha}}{3 \pi s c} \left(\frac{1}{4} - s^2 (Q_1^2 + Q_2^2)\right) + \frac{3 \tilde{\alpha} c}{4 \pi s} + \frac{\tilde{\alpha} c}{6 \pi c s} \right\} 
\]

\( (59) \)

\[
\text{div } \nabla \omega(0) = \frac{\tilde{\alpha}}{4 \pi s^2} \left\{ - \frac{5 t}{12 c^2} \Delta t - \frac{t}{12 c^2} \Delta b - \left(1 - \frac{s^2}{c^2} \right) \Delta \omega \right\}
\]

\( (60) \)

\[
\text{div } \nabla \omega(q^2) = \frac{\tilde{\alpha}}{4 \pi} \left\{ - \frac{t}{2 s^2 c^2} \Delta t + \left(4 + 1/c^2 - 1/s^2 \right) \Delta \omega \right\}
\]

\( (61) \)

\[
\text{div } \Gamma_\nu(q^2) = - \frac{\tilde{\alpha}}{4 \pi} \frac{1}{2 s^2} \Delta \omega
\]

\( (62) \)

\[
\text{div } \Gamma_\rho(q^2) = - \frac{\tilde{\alpha}}{8 \pi s^3} \Delta \omega
\]

\( (63) \)

\[
\text{div } \Gamma_\lambda(q^2) = - \frac{\tilde{\alpha}}{8 \pi s^3} \Delta \omega
\]

\( (64) \)

\[
\text{div } \Gamma_\phi(q^2) = \Delta \omega \frac{\tilde{\alpha}}{16 \pi s c} \left( \frac{1}{4 c^2 s^2} + \frac{2 s^2 - 1}{2 s^2} + \frac{3 c^2}{s^2} \right)
\]

\( (65) \)

\[
\text{div } D = \frac{\tilde{\alpha}}{4 \pi s^2} \Delta \omega
\]

\( (66) \)

\[
\text{div } \Delta_\nu = - \Delta \omega \frac{\tilde{\alpha}}{4 \pi} \left(1 + (a_c + \nu_c)^2 + 1/2 s^2 \right)
\]

\( (67) \)

Here \( \Delta t = 1/\epsilon - \gamma + \ln(m^2 / \mu^2) \). Now inserting these expressions into the expressions for \( s_{\omega}(q^2) \) and \( \omega_{\omega}(q^2) \) we see the cancellation of the divergences. Let us first analyse \( s_{\omega}(q^2) \):

\[
\text{sc div } \left\{ \Pi_{\nu \omega}(q^2) + 4 sc M_{\omega}^2 \Gamma_\nu(q^2) + 4 sc (-a_c \Gamma_\nu(q^2) + \nu_c \Gamma_\lambda(q^2) \right\}
\]

\[
= \frac{\tilde{\alpha}}{\pi} \left[ 1/4 (Q_1 - Q_2) - c^2 (Q_1^2 + Q_2^2) \right] \Delta \omega + \frac{N_c \tilde{\alpha}}{3 \pi} \left( - \frac{1}{4} + s^2 (Q_1^2 + Q_2^2) \right) \Delta \omega + \frac{\tilde{\alpha}}{4 \pi} \left( \frac{c^2}{2} + \frac{1}{6} \right) \Delta \omega
\]

\( (68) \)

where the divergent terms in \( \Pi_{\nu \omega} \) and \( \Gamma_\nu(q^2) \) cancel each other. The divergences coming from \( s_0^2 \) can be shown to lead to the expression

\[
-e^2 \text{div} \left( \Pi_{\nu \omega}(M_{\omega}^2) - \Pi_{\omega}(M_{\omega}^2) \right) = \frac{\tilde{\alpha}}{\pi} \left[ 1/4 (Q_1 - Q_2) - c^2 (Q_1^2 + Q_2^2) \right] \Delta \omega + \frac{N_c \tilde{\alpha}}{3 \pi} \left( - \frac{1}{4} + s^2 (Q_1^2 + Q_2^2) \right) \Delta \omega + \frac{\tilde{\alpha}}{4 \pi} \left( \frac{c^2}{2} + \frac{1}{6} \right) \Delta \omega.
\]

\( (69) \)

From the expression of \( s_{\omega}^2(q^2) \) we see the cancellation of the divergences explicitly.

Finally let us present the cancellation of the divergent terms in \( \omega_{\omega}(q^2) \):

\[
1/2 \text{div} (\Delta f_0 + \Delta M_{\omega} + \Pi_{\omega}(q^2)) = - \frac{\tilde{\alpha}}{2 \pi} \frac{c^2}{s^2} \Delta \omega
\]

\( (70) \)

\[
\text{div} (2 sc \Gamma_\nu(q^2) + 1/2 \Delta \nu) = \frac{\tilde{\alpha}}{4 \pi} \frac{c^2}{s^2} \Delta \omega
\]

\( (71) \)
The sum of these two gives

\[-\frac{\tilde{\alpha} c^2}{4\pi s^2} \Delta w.\]

Using also

\[2sc \text{ div } \Gamma_k^Z = -\frac{\tilde{\alpha} c^2}{4\pi s^2} \Delta w\]

we reach the explicit cancellation of all the divergences in \( q_{\nu\alpha}(q^2) \).

Hence we have illustrated the cancellation of all the divergent terms in the matrix element in (6).

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